Tomo XII 15 de Julio Núm. 78

ANALES DEL INSTITUTO DE INJENIEROS

SUMARIO: Estudios sobre desagües.—Sesiones del Directorio.—Bibliografía.

ESTUDIOS SOBRE DESAGÜES

Fórmulas jenerales i aplicaciones

Estos estudios tienen por objeto, de establecer por el cálculo analítico las fórmulas jenerales que dan en los colectores ovoides, las secciones i perímetros mojados en funcion de la altura interior del colector (h) o en funcion del radio (r) pues el radio r del colector vale $\frac{1}{3}$ de h.

Tomaré como base sucesivamente los tipos de colectores ovoides de Francfort i de Berlin, siendo el tipo de Francfort mas importante para nosotros, ya que se trata de implantar este tipo en la ejecucion de los desagües de la ciudad de Santiago.

I. Colector ovoide del tipo de Francfort

La seccion de colector (véase fig. 1) está constituida por 3 partes distintas cuyos centros son respectivamente O, O' i O''. Para calcular analíticamente

$$1.^{\circ})\ S\ (\text{seccion}) = f\ (h)\ \text{o}\ f\ (r)$$

$$2.^{\circ})\ P\ (\text{perímetro}) = F\ (h)\ \text{o}\ F\ (r)$$

establecemos las ecuaciones de los círculos de centros O, O' i O'' respecto a 2 ejes coordenados X e Y, trasportados en el punto a (véase fig. 1).

a) *Ecuacion del circulo de centro O respecto a los ejes coordenados X e Y.*

Seccion S

$$x^2+y^2=R^2 \text{ (del centro)}$$

Para el caso que nos ocupa tenemos

$$\left(x-\frac{1}{4}r\right)^2+y^2=\left(\frac{r}{4}\right)^2$$

$$y^2=\left(\frac{r}{4}\right)^2-\left(x-\frac{r}{4}\right)^2 \quad y=\pm\sqrt{\left(\frac{r}{4}\right)^2-\left(x-\frac{r}{4}\right)^2}$$

Consideramos una seccion hecha a la altura x, a partir del punto a i consideramos un elemento infinitamente pequeño de altura dx. La seccion de este elemento tendrá el valor siguiente:

$$S=y\,dx=\sqrt{\left(\frac{r}{4}\right)^2-\left(x-\frac{r}{4}\right)^2}\,dx$$

i tomando la integral desde o hasta x tendremos la seccion total

$$S_0^x=\int_0^x y\,dx=\int_0^x\sqrt{\left(\frac{r}{4}\right)^2-\left(x-\frac{r}{4}\right)^2}\,dx$$

Ponemos

$$\frac{r}{4}=a \qquad x-\frac{r}{4}=X$$

luego

$$dx=dX$$

$$S_0^x=\int_0^x\sqrt{a^2-X^2}\,dX$$

sacando la integral

$$S_0^x=\left[\frac{X}{2}\sqrt{a^2-X^2}+\frac{a^2}{2}\text{ arc sen }\frac{X}{a}+C\right]_0^x$$

Reemplazando:

$$S_0^x=\frac{x-\frac{r}{4}}{2}\sqrt{\left(\frac{r}{4}\right)^2-\left(x-\frac{r}{4}\right)^2}+\frac{1}{2}\left(\frac{r}{4}\right)^2\text{ arc sen }\frac{x-\frac{r}{4}}{\frac{r}{4}}-C$$

El valor de C se obtendrá haciendo $x=0$

$$S_0^x=\frac{x-\frac{r}{4}}{2}\sqrt{\left(\frac{r}{4}\right)^2-\left(x-\frac{r}{4}\right)^2}+\frac{1}{2}\left(\frac{r}{4}\right)^2\text{arc sen}\,\frac{x-\frac{r}{4}}{\frac{r}{4}}$$

$$-\left(\frac{r^2}{32}\times\frac{3\pi}{2}\right)$$

pues sabemos que arc sen $(-1)=\frac{3\pi}{2}$

Perímetro P. Hemos hallado el valor de:

$$y=\sqrt{\left(\frac{r}{4}\right)^2-\left(x-\frac{r}{4}\right)^2}=\left[\left(\frac{r}{4}\right)^2-\left(x-\frac{r}{4}\right)^2\right]^{1/2}$$

i por derivacion

$$\frac{dy}{dx}=y'=\frac{1}{2}\left[\left(\frac{r}{4}\right)^2-\left(x-\frac{r}{4}\right)^2\right]^{-\frac{1}{2}}-2\left(x-\frac{r}{4}\right)$$

Pero, sabemos por la Jeometría Analitica, en el capitulo que trata de la rectificacion de las curvas planas, que la lonjitud del arco comprendido entre dos puntos correspondientes a los valores o i x es la siguiente:

$$P=\int_0^x\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx=\int_0^x\sqrt{1+y'^2}\,dx$$

$$P=\int_0^x\sqrt{1+\frac{\left(x-\frac{r}{4}\right)^2}{\left(\frac{r}{4}\right)^2-\left(x-\frac{r}{4}\right)^2}}\,dx$$

$$=\int_0^x\sqrt{\frac{\left(\frac{r}{4}\right)^2}{\left(\frac{r}{4}\right)^2-\left(x-\frac{r}{4}\right)^2}}$$

Poniendo $\begin{cases} \frac{r}{4} = a \\ x - \frac{r}{4} = X \end{cases}$ $dx = dX$ tenemos

$$P_0^x = \int_0^x \sqrt{\frac{a^2}{a^2 - X^2}}\, dX = a \int_0^x \sqrt{\frac{1}{a^2 - X^2}}\, dX$$

$$P_0^x = \left[a \text{ arc sen} \frac{x - \frac{r}{4}}{\frac{r}{4}} + C \right]_0^x$$

$$= \frac{r}{4} \text{ arc sen} \frac{x - \frac{r}{4}}{\frac{r}{4}} - C \text{ Haciendo } x = o$$

tenemos $C = \frac{r}{4} \text{ arc sen} - \frac{\frac{r}{4}}{\frac{r}{4}} = \frac{r}{4} \text{ arc sen} (-1) = -\frac{r}{4} \cdot \frac{3\pi}{2}$

$$P_0^x = \frac{r}{4} \text{ arc sen} \frac{x - \frac{r}{4}}{\frac{r}{4}} - \frac{r}{4} \cdot \frac{3\pi}{2}$$

b) *Ecuacion del círculo de centro O' respecto a los mismos ejes X e Y.*

Seccion S.

Aplicando la fórmula jeneral del caso

$$(x - a)^2 + (y + b)^2 = R^2 \text{ tenemos}$$

$$(x - 2r)^2 + \left(y + \frac{5}{3} r\right)^2 = \left(\frac{8}{3} r\right)^2 \text{ aplicable}$$

desde $x = -\frac{2r}{29}$ hasta $x = 2r$

$$y=\sqrt{\left(\frac{8}{3}\,r\right)^2-(x-2r)^2}-\frac{5}{3}\,r$$

$$S_{\substack{2r\\29}}^{x}=\int_{\substack{2r\\29}}^{x} y\,dx=\int_{\substack{2r\\29}}^{x}\left[\left(\frac{8}{3}\,r\right)^2-(x-2r)^2\right]^{\frac{1}{2}}dx$$

$$-\int_{\substack{2r\\29}}^{x}\frac{5}{3}\,r\,dx$$

Poniendo $\left\{\begin{array}{l}\frac{8}{3}\,r=a\\ x-2r=X\end{array}\right.$ tenemos $dx=dX$ i

$$S_{\substack{2r\\29}}^{x}=\int_{\substack{2r\\29}}^{x} y\,dx=\int_{\substack{2r\\29}}^{x}\sqrt{a^2-X^2}-\int_{\substack{2r\\29}}^{x}\frac{5}{3}\,r\,dx$$

$$=\left[\frac{X}{2}\sqrt{a^2-X^2}+\frac{a^2}{2}\,\text{arc sen}\,\frac{X}{a}-\frac{5}{3}\,r\,x+C\right]_{\substack{2r\\29}}^{x}$$

Reemplazando

$$S_{\substack{2r\\29}}^{x}=\left[\frac{x-2r}{2}\sqrt{\left(\frac{8}{3}\,r\right)^2-(x-2r)^2}+\frac{\left(\frac{8}{3}\,r\right)^2}{2}\,\text{arc sen}\,\frac{x-2r}{\frac{8}{3}\,r}\right.$$

$$\left.-\frac{5}{3}\,r\,x+C\right]_{\substack{2r\\29}}^{x}$$

$$=\frac{x-2r}{2}\sqrt{\left(\frac{8}{3}\,r\right)^2-(x-2r)^2}+\frac{\left(\frac{8}{3}\,r\right)^2}{2}\,\text{arc sen}\,\frac{x-2r}{\frac{8}{3}\,r}$$

$$-\frac{5}{3}\,r\;.\;r\,x-C$$

Haciendo $x = \frac{2r}{29}$ tendremos el valor de C

$$C = -\frac{56r}{58}\sqrt{\frac{64}{9}r^2 - \left(\frac{2r}{29}\right)^2 - 4r^2 + 4r\frac{2r}{29}} -$$

$$C = r^2\left[\left(-\frac{56}{58}\times\frac{130}{87}\right) + \frac{32}{9}\text{ arc sen }\left(-\frac{21}{29}\right) - \frac{10}{87}\right]$$

$$C = r^2\left[-1,7756 + 3.55555\times 1.74223\pi - 0.1149\right]$$

$$C = r^2\left(6,194625\pi - 1.890606\right)$$

$$S_{\frac{2r}{29}}^{x} = \frac{x-2r}{2} \sqrt{\left(\frac{8}{3}r\right)^2 - (x-2r)^2}$$

$$+ \frac{\left(\frac{8}{3}r\right)^2}{2}\text{ arc sen }\frac{x-2r}{\frac{8}{3}r}$$

$$- \frac{5}{3}rx - r^2\left(6,194625\pi - 1.890606\right)$$

Perímetro P.—Hemos visto que

$$y = \sqrt{\left(\frac{8}{3}r\right)^2 - (x-2r)^2} - \frac{5}{3}r$$

$$= \left[\left(\frac{8}{3}r\right)^2 - (x-2r)^2\right]^{\frac{1}{2}} - \frac{5}{3}r$$

Luego $\frac{dy}{dx} = y' = \frac{1}{2}\left[\left(\frac{8}{3}r\right)^2 - (x-2r)^2\right]^{-\frac{1}{2}} - 2(x-2r)$

$$P = \int_{\frac{2r}{29}}^{x}\sqrt{1+y'^2}\,dx = \int_{\frac{2r}{29}}^{x}\sqrt{\frac{\left(\frac{8}{3}r\right)^2}{\left(\frac{8}{3}r\right)^2 - (x-2r)^2}}\,dx$$

Haciendo $\begin{cases} \frac{8}{3}\, r = a \\ x - 2r = X \end{cases}$ luego $dx = dX$ i

$$P = \int_{\frac{2r}{29}}^{x} \sqrt{1+y'^2}\, dx = \int_{\frac{2r}{29}}^{x} \sqrt{\frac{a^2}{a^2 - X^2}}\, dX$$

$$= a \int_{\frac{2r}{29}}^{x} \sqrt{\frac{1}{a^2 - X^2}}\, dX = a \text{ arc sen } \frac{X}{a} + C$$

i reemplazando

$$= \left[\frac{8}{3}\, r \times \text{arc sen } \frac{x - 2r}{\frac{8}{3}\, r} + C \right]_{\frac{2r}{29}}^{x}$$

$$= \frac{8}{3}\, r \times \text{arc sen } \frac{x - 2r}{\frac{8}{3}\, r} - C$$

Obtendremos el valor de C haciendo $x = \frac{2r}{29}$

$$P_{\frac{2r}{29}}^{x} = \frac{8}{3}\, r \times \text{arc sen } \frac{x-2r}{\frac{8}{3}\, r} - \frac{8r}{3} \text{ arc sen } \frac{\frac{2r}{29} - 2r}{\frac{8r}{3}}$$

C) *Ecuacion del circulo de radio r i del centro O'' respecto a los ejes X e Y.*

Seccion S.

La ecuacion jeneral en este caso es de la forma

$$(x - a)^2 + y^2 = R^2$$

$(x - 2r)^2 + y^2 = r^2$ aplicable desde $x = 2r$ hasta $x = 3r$

$$y^2 = r^2 - (x-2r)^2$$

$$y=\sqrt{r^2 - (x-2r)^2}$$

$$S_{2r}^{x} = \int_{2r}^{x} y\, dx = \sqrt{r^2 - (x-2r)^2}\, dx$$

Haciendo $x-2r=X$ luego $dx=dX$ i

$$S_{2r}^{x} = \int_{2r}^{x} y\, dx = \int_{2r}^{x} \sqrt{r^2 - X^2}\, dX$$

$$S_{2r}^{x} = \int_{2r}^{x} y\, dx = \frac{X}{2} \sqrt{r^2-X^2} + \frac{r^2}{2} \text{ arc sen } \frac{X}{r} + C$$

Reemplazando:

$$= \left[\frac{x-2r}{2} \sqrt{r^2-(x-2r)^2} + \frac{r^2}{2} \text{ arc sen } \frac{x-2r}{r} + C \right]_{27}^{x}$$

$$= \frac{x-2r}{2} \sqrt{r^2-(x-2r)^2} + \frac{r^2}{2} \text{ arc sen } \frac{x-2r}{r} - C$$

El valor de C se calcula haciendo $x=2\,r$ i en este caso $C=0$
Deconsiguiente:

$$S_{2r}^{x} = \frac{x-2r}{2} \sqrt{r^2(x-2r)^2} + \frac{r^2}{2} \text{ arc sen } \frac{x-2r}{r}$$

Perímetro P.—Hemos visto que

$$y = \sqrt{r^2-(x-2r)^2} \text{ i luego}$$

$$\frac{dy}{dx} = y' = \frac{1}{2} \left[r^2-(x-2r)^2 \right]^{-\frac{1}{2}} - 2\,(x-2r)$$

$$P_{2r}^{x} = \int_{2r}^{x} \sqrt{1 + y'^2}\, dx = \int_{2r}^{x} \sqrt{\frac{r^2}{r^2-(x-2r)^2}}\, dx$$

Haciendo $x-2r = X$ luego $dx = dX$ i

$$P_{2r}^{x} = \int_{2r}^{x} \sqrt{1 \times y'^2}\, dx = r \int_{2r}^{x} \sqrt{\frac{1}{r^2-X^2}}\, dX$$

$$= \left[r \text{ arc sen } \frac{X}{r} + C \right]_{2r}^{x} = \left[r \text{ arc sen } \frac{x-2r}{r} + C \right]_{2r}^{x}$$

$$= r \text{ arc sen } \frac{x-2r}{r} - C$$

$C = o$ por el mismo motivo que en el caso precedente

$$P_{2r}^{x} = \int_{2r}^{x} \sqrt{1 + y'^2}\, dx = r \text{ arc sen } \frac{x-2r}{r}$$

RESÚMEN

A) *Superficies S.*

1.° Desde h=o hasta h= $\frac{2r}{29}$

$$S_{0}^{\frac{2r}{29}} = \frac{x - \frac{r}{4}}{2} \sqrt{\left(\frac{r}{4}\right)^2 - \left(x - \frac{r}{4}\right)^2}$$

$$+ \frac{1}{2} \left(\frac{r}{4}\right)^2 \text{ arc sen } \frac{x - \frac{r}{4}}{\frac{r}{4}} - \left(\frac{r^2}{32} \frac{3\pi}{2}\right)$$

2.° Desde h=$\frac{2r}{29}$ hasta h=2 r

$$S_{\frac{2r}{29}}^{2r} = \frac{x-2r}{2} \sqrt{\left(\frac{8}{3} r\right)^2 - (x-2r)^2}$$

$$+ \frac{\left(\frac{8}{3} r\right)^2}{2} \text{ arc sen } \frac{x-2r}{\frac{8}{3} r} - \frac{5}{3} rx - r^2(6,194625\pi - 1,890606)$$

3.° Desde h=2 r hasta h=3 r

$$S_{2r}^{3r} = \frac{x-2r}{2} \sqrt{r^2-(x-2r)^2} + \frac{r^2}{2} \text{ arc sen } \frac{x-2r}{r}$$

B) *Perímetros P*

1.° Desde h=o hasta h= $\frac{2r}{29}$

$$P_0^{\frac{2r}{29}} = \frac{r}{4} \text{ arc sen } \frac{x-\frac{r}{4}}{-\frac{r}{4}} - \frac{r}{4} \frac{3\pi}{2}$$

2.° Desde h= $\frac{2r}{29}$ hasa h=2r

$$P_{\frac{2r}{29}}^{2r} = \frac{8}{3} r \text{ arc sen } \frac{x-2r}{\frac{8}{3}r} - 4{,}64596\,\pi r$$

3.° Desde h=2r hasta h=3r

$$P_{2r}^{3r} = r \text{ arc sen } \frac{x-2r}{r}$$

OBSERVACIONES

S=Mitad de la Superficie entre dos Secciones Horizontales.
P=Mitad del perímetro mojado entre las mismas secciones.
x=Altura del agua en el cauce.
r=Radio que corresponde a la seccion respectiva.

APLICACIONES

Vamos a hacer la aplicacion al tipo de Francfort buscando las superficies i perímetros mojados correspondientes a alturas que varían de 0,1r.

I. SUPERFICIES

a) Hemos visto que

$$S_0^x = S_0^{\frac{2r}{29}} = \frac{x-\frac{r}{4}}{2} \sqrt{\left(\frac{r}{4}\right)^2 - \left(x-\frac{r}{4}\right)^2}$$

$$+ \frac{1}{2}\left(\frac{r}{4}\right)^2 \text{arc sen} \frac{x-\frac{r}{4}}{\frac{r}{4}} - \frac{3\pi r^2}{64}$$

Poniendo $x=ar$ como $ar=\frac{2r}{29}$ i $a=\frac{2}{29}$

$$S_0^a = \frac{r^2}{4}\left\{\left(2a-\frac{1}{2}\right)\sqrt{\left(\frac{1}{4}\right)^2-\left(a-\frac{1}{4}\right)^2} + \frac{1}{8}\,\text{arc sen}\,(4a-1) - \frac{3\pi}{16}\right\}$$

aplicable desde $a=o$ hasta $a=\frac{2}{29}$ i la superficie del 1.er elemento cuya altura $=\frac{2r}{29} < 0{,}1r$ será

$$S_0^{\frac{2}{29}} = 0{,}0327012\,\frac{r^2}{4} = 0{,}0081753\,r^2$$

b) Tenemos en el 2.° caso

$$S_{\frac{2r}{29}}^{x} = \frac{x-2r}{2}\sqrt{\left(\frac{8}{3}\right)^2-(x-2r)^2}$$

$$+ \frac{\left(\frac{8}{3}r\right)^2}{2}\,\text{arc sen}\,\frac{x-2r}{\frac{8}{3}r} - \frac{5}{3}\,r\,x - r^2\,(6{,}194625\pi - 1.890606)$$

aplicable desde $x=\frac{2r}{29}$ hasta $x=2r$

Poniendo $x=ar$ tenemos

$$S_{\frac{2}{29}}^{a} = r^2\left\{\frac{a-2}{2}\sqrt{\left(\frac{8}{3}\right)^2-(a-2)^2} + \frac{32}{9}\,\text{arcsen}\,\frac{3}{8}(a-2) - \frac{5}{3}\,a - (6{,}194625\pi - 1.890606)\right\}$$

aplicable desde $a=\frac{2}{29}$ hasta $a=2$

Las superficies se hallan en el cuadro siguiente:

a	$a-2$	$\frac{a-2}{2}$	$(a-2)^2$	$\frac{3}{8}\left(a-2\right)$	arc sen $\frac{3}{8}\left(a-2\right)$
0.1	−1.9	$-\frac{19}{20}$	3.61	−1.9×0.375=−0.7125	1.747563 π
0.2	" 1.8	" $-\frac{18}{20}$	3.24	" 1.8×0.375= " 0.6750	1,764143 "
0.3	" 1.7	" $-\frac{17}{20}$	2.89	" 1.7×0,375= " 0,6375	1,779962 "
0.4	" 1.6	" $-\frac{16}{20}$	2.56	" 1.6×0.375= " 0.6	1,795167 "
0.5	" 1.5	" $-\frac{15}{20}$	2.25	" 1.5×0.375= " 0.5625	1,809839 "
0.6	" 1.4	" $-\frac{14}{20}$	1.96	" 1.4×0.375= " 0.525	1.824065 "
0.7	" 1.3	" $-\frac{13}{20}$	1.69	" 1 3×0.375= " 0.4875	1.837909 "
0.8	" 1.2	" $-\frac{12}{20}$	1.44	" 1.2×0.375= " 0.45	1,851423 "
0.9	" 1.1	" $-\frac{11}{20}$	1.21	" 1.1×0.375= " 0.4125	1,864656 "
1.	" 1	" $-\frac{10}{20}$	1	" 1×0.375= " 0·375	1,877643 "
1.1	" 0.9	" $-\frac{9}{20}$	0.81	" 0.9×0.375= " 0.3375	1,890419 "
1.2	" 0.8	" $-\frac{8}{20}$	0.64	" 0.8×0.375= " 0.3	1,903012 "
1.3	" 0.7	" $-\frac{7}{20}$	0.49	" 0.7×0.375= " 0.2625	1,915453 "
1.4	" 0.6	" $-\frac{6}{20}$	0,36	" 0·6×0.375= " 0.225	1,927772 "
1.5	" 0.5	" $-\frac{5}{20}$	0.25	" 0.5×0.375= " 0.1875	1,939962 "
1.6	" 0.4	" $-\frac{4}{20}$	0.16	" 0.4×0.375= " 0.15	1,952073 "
1.7	" 0.3	" $-\frac{3}{20}$	0.09	" 0.3×0.375= " 0.1125	1,964114 "
1.8	" 0.2	" $-\frac{2}{20}$	0.04	" 0.2×0.375= " 0.075	1,976105 "
1.9	" 0.1	" $-\frac{1}{20}$	0.01	" 0.1×0.375= " 0.0375	1.988061 "
2.	0	0	0	0	2 π

$\frac{a+2}{2}\sqrt{\frac{64}{9}-(a-2)^2}$	$-\frac{5}{3}a$	$\frac{32}{9}$ arc sen $\frac{3}{8}(a-2)$	Suma de los 3 últimos term.	Constante	Superficies
— 1.77757	— 0.16666	19.520511	17.576241	17.570394	0.005847r²
" 1.77076	" 0.33333	19.705717	17.601657	Id.	0.031263r²
" 1.74635	" 0.50000	19.882418	17.637068	"	0.066674r²
" 1.70666	" 0.66666	20.052255	17.678995	"	0.108601r²
" 1.65375	" 0.83333	20.216133	17.729053	"	0.158659r²
" 1.58876	" 1.00000	20.375048	17.786388	17.570394	0.215994r²
" 1.51342	" 1.16666	20.529690	17.849570	"	0.279176r²
" 1.42880	" 1.33333	20.680641	17.918541	"	0.348147r²
" 1.33595	" 1.50000	20.828456	17.993506	"	0.423112r²
" 1.23603	" 1.66666	20.973523	18.070893	"	0.500499r²
" 1.12959	" 1.83333	21,116582	18.153692	"	0.583298r²
" 1.017553	" 2.00000	21.256892	18.240338	"	0.669944r²
" 0.900603	" 2.16666	21.395863	18.328560	"	0.758166r²
" 0.770487	" 2.33333	21.533469	18.420682	"	0.850288r²
" 0.65484	" 2.50000	21.669623	18.515783	"	0.945389r²
" 0.52730	" 2.66666	21.804916	18.611016	"	1.040622r²
" 0.39745	" 2.83333	21.939414	18.708664	"	1.138270r²
" 0.265916	" 3.00000	22.073356	18.807440	"	1.237046r²
" 0.13324	" 3.16666	22.206905	18.907065	"	1.336671r²
0	" 3.33333	22.340263	19.006963	"	1.436569r²

C) En fin en el tercer caso

$$S_{2r}^{x} = \frac{x-2r}{2}\sqrt{r^2-(x-2r)^2} + \frac{r^2}{2}\text{arc sen}\frac{x-2r}{r}$$ aplicable desde

$x=2r$ hasta $x=3r$. Poniendo $x=ar$ tendremos

$$S_{r}^{a} = \frac{r^2}{2}\left\{(a-2)\sqrt{1-(a-2)^2} + \text{arc sen}\,(a-2)\right\}$$

fórmula que sirve de base para el cálculo de las distintas superficies correspondientes al tercer caso. El valor de las superficies está consignado en el cuadro siguiente:

a	$a-2$	$(a-2)^2$	$(a-2)\sqrt{1-(a-2)^2}$	arc sen$(a-2)$	SUPERFICIES
2.1	0.1	0.01	0.099499	0.1001542	0.0998266 r^2
2.2	0.2	0.04	0.19596	0·2013577	0.1986589 r^2
2.3	0.3	0.09	0.286182	0.3046944	0.2954382 r^2
2.4	0.4	0.16	0.366608	0.4115182	0.3890631 r^2
2.5	0.5	0.25	0 433015	0.5235979	0.4783065 r^2
2.6	0.6	0.36	0.48000	0.6435034	0.5617517 r^2
2.7	0.7	0.49	0.499898	0.7753971	0.6376476 r^2
2.8	0.8	0.64	0.48000	0.9273867	0.7036933 r^2
2.9	0.9	0.81	0.392301	1.1197715	0.7560362 r^2
3	1	1	0	1 5708	0·7854000 r^2

II. PERÍMETROS

a) Hemos visto que

$$P_{0}^{x} = \frac{r}{4}\left(\text{arc sen}\frac{x-\frac{r}{4}}{\frac{r}{4}} - \frac{3\pi}{2}\right)$$ Poniendo $x=ar$

$$P_{0}^{a} = r\left\{\frac{1}{4}\text{arc sen}(4a-1) - \frac{3\pi}{8}\right\}$$ aplicable

desde $a=o$ hasta $a=\frac{2}{20}$

$$P_{0}^{\frac{2}{29}} = 0.1902534\ r$$

b) tenemos en el 2.° caso

$$P_{\frac{2r}{29}}^{x} = \frac{8}{3}\ r \text{ arc sen } \frac{\frac{x-2r}{8}}{\frac{}{3}r} - 4,645966\ \pi\ r \text{ aplicable}$$

desde $x = \frac{2r}{29}$ hasta $x=2r$ Poniendo $x=ar$

$$P_{\frac{2}{29}}^{a} = r \left\{ \frac{8}{3} \text{ arc sen } \frac{3}{8} (a-2) - 4.645966\ \pi \right\}$$

Tenemos las superficies en el cuadro siguiente:

a	arc sen $\frac{3}{8}(a-2)$	$\frac{8}{3}$ arc sen $\frac{3}{8}(a-2)$	$-4,465966\ \pi$	Perímetros ó Largos
0.1	1.747563 π	14.6403838	−14,5957670	0.0446168 r
0.2	1.764143 »	14.7792865	» 14,5957670	0.1835195 r
0.3	1.779962 »	14.9118146	» 14,5957670	0.3160474 r
0.4	1.795167 »	15 0391910	» 14,5957670	0.4434240 r
0.5	1.809839 »	15.1621115	» 14,5957670	0.5663445 r
0.6	1.824065 »	15.2812859	» 14,5957670	0.6855189 r
0.7	1.837909 »	15.3972675	» 14.5957670	0.8015005 r
0.8	1.851423 »	15.5104813	» 14,5957670	0.9147143 r
0.9	1.864656 »	15.6213221	» 14,5957670	1.0255751 r
1	1.877643 »	15.7301420	» 14,5957670	1.1343750 r
1,1	1.890419 »	15.8374371	» 14,5957670	1.2416701 r
1.2	1.903012 »	15.9426681	» 14,5957670	1.3469011 r
1.3	1.915453 »	16.0468969	» 14,5957670	1.4511299 r
1.4	1.927772 »	16.1501016	» 14,5957670	1.5543346 r
1.5	1.939962 »	16.2522170	» 14,5957670	1.6564500 r
1.6	1.952073 »	16.3536867	» 14,5957670	1.7579197 r
1.7	1.964114 »	16.4545651	» 14,5957670	1.8587981 r
1.8	1.976105 »	16.5550162	» 14,5957670	1.9592492 r
1.9	1.988061 »	16.6551798	» 14,5957670	2.0594128 r
2	2 π	16.7551989	» 14,5957670	2.1594319 r

C) I en el tercer caso

$$P_{2r}^{x} = r \text{ arc sen} \frac{x-2r}{2} \text{ aplicable desde } x=2r \text{ hasta } x=3r$$

$$\text{Poniendo } x=ar \quad P_{2}^{a} = r \text{ arc sen } (a-2)$$

El cálculo de los perimetros en este caso da lugar al cuadro siguiente:

a	arc sen $(a-2)$	Perimetros o largos
2.1	0.03188 π	0.1001542r
2.2	0.064094 »	0.2013577r
2.3	0.096987 »	0.3046944r
2.4	0.13099 »	0.4115182r
2.5	0.166666 »	0.5235979r
2.6	0.204833 »	0.6435034r
2.7	0.246816 »	0.7753971r
2.8	0.295167 »	0.9273867r
2.9	0 3564335»	0.1197715r
3	0.5 π	0.5708r

TABLAS

Superficies, Perimetros i Radios medios del colector tipo de Francfort i por distintas alturas que varian de 0,1 r.

Las siguientes tablas dan las superficies los Perimetros i Radios medios del cauce completo. i de varios cauces del mismo tipo cuyo radio varía entre r=1.00m. i r=0.m.25.

Haré observar que en estas tablas, para obtener las secciones i perímetros verdaderos se ha doblado los mismos valores dados por los cuadros precedentes pues como le hice observar anteriormente en estos cuadros las superficies i perimetros solamente figuran con la mitad de su valor.

Establecido esto i tomando eu cuenta los cuadros que preceden tenemos las tablas siguientes que dan las superficies, perimetros i radios medios por todos los colectores del tipo de Francfort cuyo radio varía entre 1m. i 0.m.25 i por alturas que varían de 0, 1 r.

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a	Superficies	Perimetros	Radio medio	h	Radio medio
	S	P	$\frac{S}{P}$		m r=1.00
	Factores de r^2	Factores de r	Factores de r	m	m
0.1	0.0280446	0.4697404	0.0597023	0.10	0.0597023
0.2	0.0788766	0.7475458	0.1055275	0.20	0.1055275
0.3	0.1496986	1.0126016	0.1478357	0.30	0.1478357
0.4	0.2335526	1.2673548	0.18428357	0.40	0.18428357
0,5	0.3336686	1.5131958	0.2205058	0,50	0.2205058
0.6	0.4883386	1.7515446	0.2559676	0.60	0.2559676
0.7	0.5747026	1 9835078	0.2897405	0.70	0.2897405
0.8	0.7126446	2.2099354	0.322473	0.80	0.322473
0.9	0.8625746	2.4316570	0.3555447	0.90	0.3555447
1.0	1.0173486	2.6492568	0.3840120	1.00	0.3840120
1.1	1,1829466	2.8638470	0.4130622	1.10	0.4130622
1.2	1.3562386	3,0743090	0.4411527	1.20	0.4411527
1,3	1.5326826	3.2827666	0.4668871	1.30	0.4668871
1.4	1·7169266	3.4891760	0.4920723	1.40	0.4920723
1.5	1,9071286	3.6934068	0.5164605	1.50	0.5164605
1.6	2,0975946	3.8963462	0.5383591	1.60	0.5383591
1.7	2.2928006	4.0981030	0.5595005	1.70	0.5595005
1.8	2.4904426	4.2990052	0.5793068	1.80	0.5793068
1.9	2.6896026	4.4993324	0.5977983	1.90	0.5977983
2.0	2.8894886	4.6993706	0.6148670	2.00	0.6148670
2.1	3.0891418	4.8996790	0.6304784	2.10	0.6304784
2.2	3.2868064	5.1020860	0.6442086	2.20	0.6442086
2.3	3.4803650	5.3087594	0.6555887	2.30	0.6555887
2,4	3.6676148	5.5224070	0.6641333	2.40	0.6641333
2.5	3.8461016	5.7465664	0.6692869	2.50	0.6692869
2.6	4.0129920	5.9863774	0.670354	2.60	0.670354
2.7	4.1647838	6.2501648	0.6663.76	2.70	0.6663476
2.8	4.2968752	6.5541440	0.6555964	2.80	0.6555964
2.9	4.4015610	6.9389136	0.6343300	2.90	0.6343300
3	4.4602886	7.8409706	0.5688439	3.00	0.5688439

h	Radio medio	h	Radio medio	h	Radio medio
r=0.90 m		r=0.85 m		r=0.80 m	
m	m	m	m	m	m
0.09	0.05373207	0.085	0.050746955	0.08	0.04776184
0.18	0.09497475	0.17	0.089698375	0.16	0.08442200
0.27	0.13305213	0.255	0.125660344	0.24	0.11826856
0.36	0.16588515	0.34	0.156640975	0.32	0.14742680
0.45	0.19845522	0.425	0.187429930	0.40	0.17640464
0.54	0.23037084	0.510	0.21757246	0.48	0.20477408
0.63	0.26076345	0.595	0.24627942	0.56	0.2317924
0.72	0.29022570	0.680	0.27410205	0.64	0.2579784
0.81	0.31999023	0.765	0.30224209	0.72	0.28443576
0.90	0.34561161	0.850	0.32641097	0.80	0.30724032
0.99	0.37175598	0.935	0.35110287	0.88	0.33014976
1.08	0.39703743	1.020	0.37497980	0.96	0.35292246
1.17	0.42019839	1.105	0.39685404	1.04	0.37350968
1.26	0.44256507	1.190	0.41826146	1.12	0.39365784
1.35	0.46481445	1.275	0.43809143	1.20	0.41316840
1.44	0.48452319	1.360	0.45760524	1.28	0.43068728
1.53	0.50355045	1.445	0.47557543	1.36	0.44760040
1.62	0.52137612	1.530	0.40241078	1.44	0.46344544
1.71	0.53801847	1.615	0.50812856	1.52	0.47823864
1.80	0.55338030	1.700	0.52263695	1.60	0.49189360
1.89	0.56743056	1.785	0.53590664	1.68	0.50438272
1.98	0.57978774	1.870	0.54757731	1.76	0.51536688
2.07	0.59002983	1.955	0.55725040	1.84	0.52447096
2.16	0.59771997	2.040	0.56451331	1.92	0.53130664
2.25	0.60235821	2.125	0.5688938	2·00	0.53542952
2.34	0.6033186	2.210	0.5698009	2.08	0.5362832
2.43	0.59971284	2.295	0.56639546	2.16	0.53307808
2.52	0.59003676	2.380	0.55725694	2.24	0.52447712
2.61	0.57089700	2.465	0.53918050	2.32	0.50746400
2.70	0.51195951	2.550	0.48351732	2.40	0.45507512

h	Radio medio	h	Radio medio
m r=0,70		m r=0,60	
m	m	m	m
0.07	0.04179161	0.06	0.03582138
0.14	0.07386925	0.12	0.06331650
0.21	0.10348499	0.18	0.08870142
0.28	0.12899845	0.24	0.11057010
0.35	0.15435406	0.30	0.13230348
0.42	0.17917732	0.36	0.15358036
0.49	0.20281835	0.42	0.17384430
0.56	0.22573110	0.48	0.19348380
0.63	0.24888129	0.54	0.21332682
0.70	0.26880903	0.60	0.23040774
0.77	0.28914354	0.66	0.24783732
0.84	0.30880689	0.72	0.26469162
0.91	0.32682097	0.78	0.28013226
0.98	0.34445061	0.84	0.29524338
1.05	0.36152235	0·90	0.30987630
1.12	0.37685137	0.96	0.32301546
1.19	0.39165035	1.02	0.33570030
1.26	0 40551476	1.08	0.34758408
1.33	0.41845881	1.14	0·35867898
1.40	0.43040690	1.20	0.36892020
1.47	0.44133488	1.26	0 37828704
1.54	0.45094602	1.32	0.38652516
1.61	0.45891209	1.38	0.39235322
1.68	0.46489331	1.44	0.3984799
1.75	0.46850083	1.50	0.40157214
1.82	0.4692478	1·56	0.4022124
1.89	0.46644332	1.62	0.39980856
1.96	0.45891748	1.68	0.39335784
2.03	0.44403100	1.74	0.3805980
2.10	0.39819073	1.80	0.34130634

h	Radio medio	h	Radio medio	h	Radio medio
	m r=0,50		m r=0,33		m r=0.25
m	m	m	m	m	m
0·05	0.0298512	0.033	0.019701759	0.025	0.0149256
0·10	0.0527638	0.066	0.034824075	0.050	0.0263819
0.15	0.0739179	0.099	0.048785781	0.075	0.0369589
0.20	0.0921418	0.132	0.060813555	0.100	0.0460709
0.25	0.1102529	0.165	0.072766914	0.125	0.0551265
0.30	0.1279838	0.198	0.084469308	0.150	0.0639919
0.35	0.1448703	0.231	0.095614365	0.175	0.0724352
0.40	0.1612365	0.264	0.10641609	0.200	0.0806183
0.45	0.1777723	0.297	0.117329751	0.225	0.0888867
0.50	0.1920065	0.330	0.126724257	0.250	0.960033
0.55	0.2065311	0.363	0·136310526	0.275	0.1032656
0.60	0.2205764	0.396	0.145580391	0.300	0.1102882
0.65	0.2334436	0.429	0.154072743	0.325	0.1167218
0.70	0.2460362	0.462	0·162383859	0.350	0.1230181
0·75	0.2582503	0.495	0.170431965	0.375	0.1291152
0.80	0.2691796	0.528	0.177658503	0.400	0.1345898
0.85	0.2797503	0.561	0.184635165	0.425	0.1398752
0.90	0.2896534	0.594	0.191171244	0.450	0.1448267
0.95	0.2988992	0.627	0.197273439	0.475	0.1494496
1.00	0.3074335	0·660	0.20290611	0.500	0.1537168
1.05	0.3152392	0.693	0.208057872	0 525	0.1576196
1.10	0.3221043	0.726	0.212588838	0.550	0.1610522
1.15	0.3277944	0.759	0.216344271	0.575	0.1638972
1.20	0.3320666	0.792	0.219163989	0.600	0.1666333
1.25	0.3346434	0.825	0.220864677	0.625	0.1673217
1.30	0·3351770	0.858	0.22121682	0.650	0.1675885
1.35	0.3331738	0.891	0.219894708	0·675	0.1665869
1.40	0.3277982	0.924	0.216346812	0.700	0.1638991
1.45	0.3171650	0.957	0.20932890	0.725	0.1585825
1.50	0.2844220	0.990	0.187718487	0.750	0.1422110

a=h	Superficie	h	Superficie
r=1.00 m		r=0.90 m	
m	m^2	m	m^2
0.1	0.0280446	0.09	0,022716126
0.2	0.0788766	0.18	0,063890048
0.3	0.1496986	0.27	0.121255866
0.4	0.2335526	0.36	0.189177606
0.5	0.3336686	0.45	0,270271566
0.6	0.4483386	0·54	0.363154266
0.7	0.5747026	0,63	0.465509106
0.8	0.7126446	0,72	0,577242126
0,9	0.8625746	0.81	0,698685426
1.0	1.0173486	0.90	0,824052365
1.1	1.1829466	0.99	0,958096746
1.2	1.3562386	1.08	1,098553266
1.3	1.5326826	1,17	1,241472906
1.4	1.7169266	1,26	1,390710546
1.5	1.9171286	1,35	1,544774166
1.6	2.0975946	1.44	1·699051626
1.7	2.2928906	1.53	1,857241386
1.8	2.4904426	1.62	2,017258506
1.9	2.6896926	1.71	2,178651006
2.0	2.8894886	1.80	2.340485766
2.1	3.0891418	1.89	2.502204858
2.2	3.2868064	1.98	2.662313184
2.3	3.4803650	2.07	2.819095650
2.4	3.6676148	2.16	2.970767988
2.5	3.8461013	2.25	3.115342296
2.6	4.0129020	2,34	3.25052352
2·7	4.1647838	2.43	3.373474878
2.8	4.2968752	2.52	3.480468912
2.9	4.4015610	2.61	3.565264410
3.	4.4602886	2.70	3.612833766

h	Superficie	h	Superficie	h	Superficie
r=08.5 m		r=0.80 m		r=0,70 m	
m	m	m	m^2	m	m^2
0.085	0.0202622235	0.08	0.017948544	0,07	0,013741854
0.170	0.0569883485	0·16	0.050481024	0.14	0.038649534
0·255	0.1081572385	0.24	0.095807104	0,21	0,073352314
0.340	9.1687417535	0.32	0.149473664	0,28	0,114440774
0.425	0.2410755635	0.40	0.213547904	0,35	0.163497614
0.510	0.3239246385	0.48	0.286936704	0,42	0,219685914
0.595	0.4152226285	0.56	0.367809664	0,49	0,281604274
0.680	0.5148857235	0.64	0.456092544	0,56	0,349195854
0.765	0.6232101485	0.72	0.552047744	0.63	0.423561554
0.850	0.7350343635	0.80	0.651103104	0.70	0.498500814
0.935	0.8546789185	0.88	0.757085824	0,77	0.579643834
1.020	0.9798823885	0.96	0.867992704	0,84	0.664556914
1.105	1.1073631785	1.04	0.980916864	0,91	0,751014474
1.190	1.2404794685	1.12	1.098833024	0,98	0,841294034
1.275	1,3779004135	1.20	1.220562304	1.05	0.934493014
1.360	1.5155120985	1.28	1.342460544	1,12	1,027821354
1.445	1.6506134585	1.36	1.467449984	1,19	1,123516394
1.530	1.7993447785	1.44	1.593883264	1,26	1.220316874
1.615	1.9433029035	1.52	1.721403264	1,33	1,317949374
1.700	2.0876555135	1.60	1.849272704	1,40	1,415849444
1.785	2.2319049505	1.68	1.977050752	1,47	1.513679482
1.870	2.374717624	1.76	2,103556096	1,54	1.610535136
1.955	2.5145637125	1.84	2,227433600	1,61	1.70537885
2.040	2.6498516093	1.92	2,347273472	1,68	1,797131252
2·125	2.778808406	2.00	2,461505024	1,75	1.884589784
2.210	2.89938672	2.08	2,56831488	1,82	1,96636608
2.295	3.0090562955	2.16	2,665461632	1,89	2.040744062
2.380	3.104492332	2.24	2,750000128	1,96	2,105468848
2.465	3.1801278225	2.32	2,816999040	2,03	2,15676489
2.550	3.2225585135	2.40	2,854584704	2.10	2,185541414

h	Superficies	h	Superficies
r=0.60 m		r=0.50 m	
m	m²	m	m²
0,06	0.010096056	0,05	0,00701115
0,12	0,028395576	0,10	0,01971915
0,18	0,053891496	0,15	0.03742465
0,24	0,084078936	0,20	0,05838815
0,30	0,120120696	0,25	0,0834175
0,36	0.161401896	0,30	0,11208465
0,42	0,206892936	0,35	0,14367565
0,48	0,256552056	0,40	0,17816115
0,54	0,310526856	0.45	0,21564365
0,60	0,366245896	0.50	0,25433715
0,66	0.425860776	0,55	0,29573665
0,72	0,448245896	0,60	0,33905965
0,78	0,551765736	0,65	0,38317065
0,84	0,618093576	0,70	0,42923165
0,90	0,686566296	0,75	0,47678215
0,96	0.755134056	0,80	0,52439865
1,02	0,825440616	0,85	0,57322265
1,08	0,896559336	0,90	0.62261065
1,14	0,968289336	0,95	0.67242315
1,20	1,040215896	1,00	0,72237215
1,26	1.112091048	1.05	0,77237215
1,32	1,183250304	1,10	0,8217016
1,38	1,252931400	1,15	0,87099125
1,44	1.320341328	1,20	0.9169037
1,50	1.384596576	1,25	0.9615254
1,56	1,44467712	1,30	1,00324800
1,62	1.499322168	1,35	1,041119595
1,68	1,546875072	1,40	1.0742188
1,74	1,58456196	1,45	1,10039025
1,80	1,605703896	1,50	1,11507215

h	Superficies	h	Superficies
r=0,33 m		r=0,25 m	
m	m²	m	m²
0.033	0.0030540569	0.025	0.0017527875
0.066	0.0085896617	0.050	0.0049297875
0.099	0.016302177	0.075	0.0093561625
0.132	0.025433878	0.100	0.0145970375
0.165	0.0363365105	0.125	0.0208542875
0.198	0.04882407	0.15	0.0280211625
0.231	0.062585113	0.175	0.0359189125
0.264	0.07760699	0.200	0.0445402875
0.297	0.093943373	0.225	0.0539109125
0.330	0.110789262	0.25	0.0635842875
0.363	0.128822884	0,275	0.0739341625
0.396	0.147694383	0.30	0.0847649125
0.429	0.166909135	0.325	0.0957926625
0.462	0.1869733067	0.35	0.1073079125
0.495	0.207686304	0.375	0.1191955375
0.528	0.22842805	0·40	0.1310996625
0.561	0.24969578	0.425	0.1433056625
0.594	0.271209199	0.45	0.1556526625
0.627	0.29290752	0.475	0.1681057875
0.660	0.314665308	0.50	0.1805930375
0.693	0·336407542	0.525	0.1930713125
0.726	0.357933216	0.55	0.2054254
0.759	0.379011748	0.575	0.2175228125
0.792	0·399403251	0.60	0.229225925
0.825	0.418840464	0.625	0.24038135
0·858	0.437014828	0.65	0.25081200
0.891	0.453514955	0.675	0.2602798987
0.924	0.467929709	0.70	0.2685547
0.957	0.47932999	0.725	0.2750975625
0.990	0.485725428	0.75	0.2787680375

a	h	Perímetros	h	Perímetros
	m r=1,00		m r=0.90	
	m	m	m	m
0.1	0.10	0.4697404	0.09	0.42276636
0.2	0.20	0.7475458	0.18	0.67279122
0.3	0.30	1.0126016	0.27	0.91134144
0.4	0.40	1.2673548	0.36	1.14061932
0.5	0.50	1.5131958	0,45	1.36187622
0.6	0·60	1.7515446	0.54	1.57639014
0.7	0,70	1.9835078	0.63	1.78515702
0.8	0.80	2.2099354	0.72	1.98894186
0.9	0.90	2.4316570	0.81	2.18849130
1	1.00	2.6492568	0.90	2.38433112
1.1	1.10	2.8638470	0.99	2.57746230
1.2	1.20	3.0743090	1.08	2.76687810
1.3	1.30	3.2827666	1.17	2.95448994
1.4	1.40	3.4891760	1.26	3.14025840
1.5	1.50	3.6934068	1.35	3.32406612
1.6	1.60	3.8963462	1.44	3.50671158
1·7	1.70	4.0981030	1.53	3,68829270
1.8	1.80	4.2990052	1.62	3.86910468
1.9	1.90	4.4993324	1.71	4.04939916
2	2.00	4.6993706	1.80	4.22943354
2.1	2·10	4.8996790	1.89	4.40971110
2.2	2.20	5.1020860	1.98	4.59187740
2.3	2.30	5.3087594	2.07	4.77788346
2.4	2.40	5.5224070	2.16	4.97016630
2.5	2.50	5.7465664	2.25	5.17190976
2.6	2.60	5.9863774	2.34	5.38773966
2.7	2.70	6.2501648	2.43	5.62514832
2.8	2.80	6.5541440	2.52	5.89872960
2.9	2.90	6.9389136	2.61	6.24502224
3	3.00	7.8409706	2.70	7,05687354

h	Perímetros	h	Perímetros	h	Perímetros
r=0.85 m		r=0,80 m		r=0,70 m	
m	m	m	m	m	m
0.085	0.399279340	0,08	0,37579232	0.07	0.32881828
0.17	0.635413930	0.16	0,59803664	0,14	0,52328206
0.255	0.860711360	0,24	0,81008128	0,21	0,70882112
0.340	1.077251580	0,32	1,01388384	0,28	0.88714836
0.425	1.286216430	0.40	1,21055664	0,35	1,05923706
0.510	1.488812910	0,48	1,40123568	0.42	1,22608122
0.595	1.685981630	0,56	1,58680624	0,49	1,38845546
0.680	1.87845090	0,64	1,76794832	0,56	1,54695478
0.765	2.066908450	0,72	1,94532560	0,63	1,70215990
0.850	2.251868280	0.80	2,11940544	0,70	1,85447976
0.935	2.43426995	0,88	2,29107760	0.77	2,00469290
1.920	2.61316265	0,96	2,45944720	0.84	2,15201630
1.105	2.79035161	1.04	2,62621328	0,91	2,29793662
1.190	2.96579960	1,12	2,79134080	0,98	2,44242320
1.275	3.13939578	1,20	2,95472544	1,05	2,58538476
1.360	3.31189427	1.28	3,11707696	1,12	2.72744234
1.445	3.48338755	1,36	3,27848240	1,19	2.86867210
1.530	3.65415442	1,44	3.43920416	1.26	3,00930364
1,615	3.82443254	1,52	3,59946592	1,33	3,14953268
1.700	4.00436501	1.60	3,76939648	1,40	3,28955942
1.785	4.16472715	1,68	3,91974320	1.47	3,42977530
1.870	4.33677310	1,76	4,08166880	1.54	3,57146020
1.955	4.51244549	1.84	4,26700752	1,61	3.71613158
2.040	4.69404595	1,92	4,41792560	1,68	3.86568490
2.125	4.88458144	2,00	4,59725312	1,75	4,02259648
2.210	5.08842079	2.08	4.78910192	1,82	4,19046418
2.295	5.31264008	2,16	5,00013484	1,89	4.37511536
2.380	5.57102240	2,24	5.24331520	1,96	4,58790080
2.465	5.89807656	2,32	5,55413088	2,03	4.85723952
2.550	6.66482501	2,40	6,27277648	2,10	5,48867942

h	Perímetros	h	Perímetros
r=0,60 m		r=0.50 m	
m	m	m	m
0.06	0.28184424	0.05	0.23487020
0.12	0.44852748	0.10	0.37377290
0.18	0.60756096	0.15	0.50630080
0.24	0.76041288	0.20	0.63367740
0.30	0.90791748	0.25	0.75659790
0.36	1.05092676	0.30	0.87577230
0.42	1.19010468	0.35	0.99175390
0.48	1.32596124	0.40	1.10493770
0.54	1.45899420	0.45	1.21582850
0.60	1.58955408	0.50	1.32462840
0.66	1.71830820	0.55	1.43192350
0.72	1.84458540	0.60	1.53715450
0.78	1.96965993	0.65	1.64138330
0.84	2.09350560	0.70	1.74458900
0.90	2.216004408	0.75	1.84670340
0.96	2.33780772	0.80	1.94817310
1.02	2.45886180	0.85	2.04905150
1.08	2.57940312	0.90	2.14950260
1.14	2.69959944	0.95	2.24966620
1.20	2.81952236	1.00	2.34968530
1.26	2.93980740	1.05	2.44983950
1.32	3.05125160	1.10	2.55104300
1.38	3.18525564	1.15	2.65437970
1.44	3.31344420	1.20	2.76120350
1.50	3.44793984	1.25	2.87328320
1.56	3.59182644	1.30	2.99318870
1.62	3.75009888	1.35	3.12508240
1.68	3.93248640	1.40	3.27797200
1.74	4.16334816	1.45	3.46945680
1.80	4.70467236	1.50	3.92048530

h	Perímetros	h	Perímetros
m r=0.33		m r=0,25	
m	m	m	m
0.033	0.155014332	0.025	0.11743510
0.066	0.246690114	0.050	0.18688645
0.099	0.334158528	0 075	0.25315040
0.132	0.418227084	0.100	0.31683870
0.165	0.499354614	0.125	0.37829875
0.198	0.578009718	0.150	0.43788615
0.231	0.654557574	0.175	0.49587695
0.264	0.729278682	0.200	0.55248385
0.297	0.80214681	0.225	0.60791425
0.330	0.874254744	0.250	0.66231420
0.363	0.94506951	0 275	0.71596175
0.396	1.01452197	0.300	0.76857725
0.429	1.083312978	0.325	0.82069165
0.462	1.15142808	0.350	0.87229400
0.495	1.218824244	0.375	0.92335170
0.528	1.285794246	0.400	0.97408655
0.561	1.35237399	0.425	1·02452575
0.594	1.418671716	0.450	1·07475130
0.627	1.484779692	0,475	1·12483310
0.660	1,550792298	0.500	1·17484265
0.693	1.61689407	0.525	1·22491975
0.726	1.68368838	0.550	1·27552150
0.759	1.751890602	0.575	1·32718985
0.792	1.82239434	0.600	1·38060175
0,825	1.896366912	0.625	1·43664160
0.858	1.975504542	0.650	1·49659435
0.891	2.062554384	0.675	1·56254120
0.924	2.16286752	0.700	1·63853600
0.957	2.280841488	0.725	1·73472840
0.990	2.587569798	0.750	1·96021265

Ejemplo. - Para indicar mejor el sentido práctico de las tablas que preceden tomaré el ejemplo concreto de un colector, tipo de Francfort, de $1^{m}.50 \times 1^{m}.00$ de Seccion. Supongamos el agua hasta la altura de 2r es decir hasta el nacimiento de la bóveda en medio punto.

El radio$=0^{m}.50$ i $2r=1^{m}.00$

Vemos segun las tablas, que, en el caso que nos ocupa, S (superficie)=0.72237215, P (perímetro mojado)=2.34968530 i R (radio medio)=0.3074335.

Poniendo estos valores en las fórmulas de los colectores ovoides $\frac{Ri}{u^2} = A$ i $Q=Su$ en que R=radio medio i=pendiente por $^{m}/_{m}$ u=velocidad media $A=0.00015\left(1+\frac{0.03}{R}\right)$ Q=Gasto por segundo S=Superficie tendremos fácilmente i rápidamente los valores de u i de Q.

La superficie de estuco i el cubo de albañilería se hallan tambien fácilmente por medio de las tablas. Calculamos por 1 metro de lonjitud.

1.°) *Estuco interior.* Este estuco va desde el fondo del colector hasta la altura de $1^{m}.00$. El radio $r=0^{m}.5075$, i segun las tablas la superficie que corresponde al estuco interior por cada metro de lonjitud$=4.6993706 \times r \times 1=4.6993706 \times 0^{m}.5075 \times 1=2^{m^2}.3849305795$.

2.°) *Albañilería de ladrillos.*

a) Albañilería de ladrillos de 0 hasta altura de 1.00. Aplicando las tablas vemos que el cubo de albañilería de ladrillos correspondiente$=4.6993706 \times r \times 0.20 \times 1 = 4.6993706 \times 0.615 \times 0.20 \times 1$ $=0^{m^3}.5780225838$ por cada metro de lonjitud ménos la parte correspondiente al concreto que se calculará aparte o bien $0^{m^3}.5780225830$ $-0^{m^3}.172=0^{m^3}.4060225838$.

b) *Albañilería de ladrillos desde $1^{m}.00$ hasta $1^{m}.50$.*

El cubo correspondiente$=(7.8409706-4.6993706) \times r \times 0^{m}.20 \times 1$ $=3.1416 \times 0^{m}.60 \times 0.20 \times 1=0^{m^3}.376992$.

c) *Cubo total de albañilería de ladrillos* por $^m/_c$. es el siguiente:

$$0.4060225838+0.376992=0.7830145838 \; m^3$$

3.°) *Estuco al estrados*—.Va desde 1.00^m hasta 1.50^m de altura. La superficie correspondiente$=3.1416\times r\times 1=3.1416\times 0.7075\times 1=2.222682$. La superficie total de estuco del colector por $^m/_c$ $=2.3849305795+2.222682=4.6076125795$.

4.°) *Concreto.*—En fin el cubo total de concreto ocupado por el colector por $^m/_c$ seria $4.4602886\times r^2\times 1=4.4602886\times \overline{0.715}^2\times 1$ mas la parte del concreto que se halla debajo de la linea curva dibujada de punto.

Santiago, Mayo de 1897.

FR. DE SUTTER.

CAPÍTULO II

II. SEGUNDO PERFIL

Perfil tipo de Berlin

La fig. (3) indica este perfil tipo—La seccion como la del tipo de Francfort está constituida por 3 partes distintas cuyos centros son respectivamente O, O' i O''.

Para calcular analíticamente

1.°) S (seccion) = f (h) o f (r)

2.°) P (perimetro) = f (h) o f (r) establecemos las ecuaciones de los circulos de los distintos centros tal como en el 1.er perfil.

I $\left(x-\frac{1}{2}r\right)^2+y^2=\frac{r^2}{4}$ aplicable desde $x=o$ hasta $x=\frac{1}{5}r$

II. $(x-2r)^2+(y+2r)^2=9r^2$ aplicable desde $x=\frac{1}{5}r$ hasta $x=2r$

III. $(x-2r)^2+y^2=r^2$ aplicable desde $x=2r$ hasta $x=3r$

Las ecuaciones se hallarán por induccion al comparar con las fórmulas anteriormente establecidas. La ecuacion I se calculará reemplazando $\frac{r}{4}$ por $\frac{r}{2}$ en la ecuacion que da el valor de S_0^x del tipo de Francfort. Tenemos

1.°) *Seccion*

$$S_0^x = \frac{x-\frac{r}{2}}{2}\sqrt{\left(\frac{r}{2}\right)^2-\left(x-\frac{r}{2}\right)^2}+\frac{1}{2}\left(\frac{r}{2}\right)^2 \text{arc sen}\frac{x-\frac{r}{2}}{\frac{r}{2}}+C$$

C=o para x=o i luego $C=-\frac{3\pi}{16}r^2$

$$S_0^x = \frac{x-\frac{r}{2}}{2}\sqrt{\left(\frac{r}{2}\right)^2-\left(x-\frac{r}{2}\right)^2}+\frac{1}{2}\left(\frac{r}{2}\right)^2 \text{arc sen}\frac{x-\frac{r}{2}}{\frac{r}{2}}$$

$$-\frac{3\pi}{16}r^2$$

Perimetro P. —

Tenemos el perímetro P reemplazando en la ecuacion P_0^x del perfil de Francfort $\frac{r}{1}$ por $\frac{r}{2}$

$$P_0^x = \frac{r}{2} \text{arc sen}\frac{x-\frac{r}{2}}{\frac{r}{2}} - \frac{r}{2}\,\frac{3\pi}{2}$$

b) La ecuacion II arriba referida se hallará reemplazando en la ecuacion correspondiente del perfil número (I) $\frac{5}{3}$ r por 2r. i $\frac{8}{3}$ r por 3r. Tenemos

1) *Seccion.*

$$S_{\frac{1}{5}r}^x = \frac{x-2r}{2}\sqrt{9r^2-(x-2r)^2}+\frac{9r^2}{2}\text{arc sen}\frac{x-2r}{3r}-2rx+C$$

Haciendo $x = \frac{1}{5} r$ tenemos el valor de C pues,

$$C = -\left(\frac{\frac{1}{5} r - 2r}{2} \sqrt{9r^2 - \left(\frac{1}{5} r - 2r\right)^2} \right.$$

$$\left. + \frac{9r^2}{2} \text{ arc sen } \frac{\frac{1}{5} r - 2r}{3r} - 2r \frac{r}{5} \right)$$

$$= -\left\{ -\frac{9r^2}{10} \sqrt{9 - \frac{81}{25}} + \frac{9r^2}{2} \text{ arc sen} \left(-\frac{3}{5}\right) - \frac{2r^2}{5} \right\}$$

$$= -r^2 \left(4.5 \times 1.7952\pi - \frac{r}{5} - \frac{108}{50}\right) = -r^2 \left(8.0784\pi - 2.56\right)$$

luego

$$S_{\frac{1}{5} r}^{x} = \frac{x - 2r}{2} \sqrt{9r^2 - (x - 2r)^2} + \frac{9r^2}{2} \text{ arc sen } \frac{x - 2r}{3r} -$$

$$- 2rx - r^2 (8.0784\pi - 2.56)$$

Perímetro P.—Reemplazando $\frac{5}{3}$ r por 2r, i $\frac{8}{3}$ r por 3r en la fórmula correspondiente del primer perfil tenemos

$$P_{\frac{1}{5} r}^{x} = 3r\left(\text{arc sen } \frac{x - 2r}{3r} - 1.7952\pi\right)$$

c) La ecuacion III. es idéntica a la correspondiente del mismo caso del perfil 1.° i tenemos los mismos valores de S i de P.

$$S_{2r}^{0} = \frac{x - 2r}{2} \sqrt{r^2 - (x - 2r)^2} + \frac{r^2}{2} \text{arc sen } \frac{x - 2r}{r}$$

$$P_{2r}^{x} = r \text{ arc sen } \frac{x - 2r}{r}$$

RESÚMEN

A) *Superficies S.*

1.° Desde h=o hasta h= $\frac{1}{5}$r

$$S_0^{\frac{1}{5}r}=\frac{x-\frac{r}{2}}{2}\sqrt{\left(\frac{r}{2}\right)^2-\left(x-\frac{r}{2}\right)^2}$$

$$+\frac{1}{2}\left(\frac{r}{2}\right)^2\text{arc sen}\frac{x-\frac{r}{2}}{\frac{r}{2}}-\frac{3\pi}{16}r^2$$

2.° Desde h= $\frac{1}{5}$ r hasta h=2r

$$S_{\frac{1}{5}r}^{2r}=\frac{x-2r}{2}\sqrt{9r^2-(x-2r)^2}+\frac{9r^2}{2}\text{arc sen}\frac{x-2r}{3}$$

$$-2r\,x-r^2(8.0784\pi-2.56)$$

3.° Desde h=2r hasta h=3r

$$S_{2r}^{3r}=\frac{x-2r}{2}\sqrt{r^2-(x-2r)^2}+\frac{r^2}{2}\text{arc sen}\frac{x-2r}{r}$$

B) *Perímetros P.*

1.° Desde h=o hasta h= $\frac{1}{5}$ r

$$P_0^{\frac{1}{5}r}=\frac{r}{2}\text{arc sen}\frac{x-\frac{r}{2}}{\frac{r}{2}}-\frac{r}{2}\frac{3\pi}{2}$$

2.° Desde $h=\frac{1}{5}r$ hasta $h=2r$

$$P^{2r}_{\frac{1}{5}r}=3r\left(\text{arc sen}\,\frac{x-2r}{3r}\quad-1,7952\pi\right)$$

3.° Desde $h=2r$ hasta $3r$

$$P^{3r}_{2r}=r\,\text{arc sen}\,\frac{x-2r}{r}$$

N. B. Las observaciones hechas en el 1[er] perfil son tambien aplicables en este caso.

APLICACIONES

Aplicamos las fórmulas precedentes al tipo de colector de Berlin i buscamos las superficies i perímetros mojados correspondientes a alturas que varian de $a=0,1r$.

I Superficies.

Hemos visto que:

a) $$S_0^{\frac{1}{5}r}=\frac{x-\frac{r}{2}}{2}\sqrt{\left(\frac{r}{2}\right)^2-\left(x-\frac{r}{2}\right)^2}$$

$$+\frac{1}{2}\left(\frac{r}{2}\right)^2\text{arc sen}\,\frac{x-\frac{r}{2}}{\frac{r}{2}}-\frac{3\pi}{16}r^2$$

Poniendo $x=a\,r$

$$S_0^{\frac{1}{5}a}=\frac{r^2}{2}\left\{\left(a-\frac{1}{2}\right)\sqrt{\frac{1}{4}-\left(a-\frac{1}{2}\right)^2}\right.$$

$$\left.+\frac{1}{4}\,\text{arc sen}\,2\left(a-\frac{1}{2}\right)-\frac{3\pi}{8}\right\}$$

aplicable desde $a=o$ hasta $a=\frac{1}{5}$

El cuadro siguiente dá las Superficies correspondientes.

a	$a-\frac{1}{2}$	$(a-\frac{1}{2})^2$	$2(a-\frac{1}{2})$	$(a-\frac{1}{2})\sqrt{\frac{1}{4}-(a-\frac{1}{4})^2}$
0.1	—0.4	0.16	—0.8	—0.12
0.2	—0.3	0.09	—0.6	—0.12

$\frac{1}{4}$ arc sen$2(a-\frac{1}{2})$	Suma de las 2 últimas cantidades	Constante	Superficies
0.42643π	1.21967	—1.1781	0,04157 $\frac{r^2}{2}$
0.448705»	1.289925	- 1.1781	0.111834 $\frac{r^2}{2}$

b) Tenemos en el 2.° caso

$$S_{\frac{1}{5}r}^{x} = \frac{x-2r}{2}\sqrt{9r^2-(x-2r)^2}+\frac{9r^2}{2}\text{arc sen}\frac{x-2r}{3r}$$

$$-2r\,x-r^2(8.0784\pi-2,56)$$

Poniendo $x=ar$

$$S_{\frac{1}{5}}^{a}=\frac{r^2}{2}\left\{(a-2)\sqrt{9-(a-2)^2}+9\ \text{arc sen}\frac{(a-2)}{3}-4a\right.$$

$$\left.-(16,1568\pi-5.12)\right\}$$

Esta fórmula da lugar al cuadro siguiente, en el cual están consignadas las superficies desde $\frac{1}{5}$r hasta 2r.

a	$a-2$	$(a-2)^2$	$\frac{a-2}{3}$	$-4\ a$	$(a-2)\sqrt{9-(a-2)^2}$
0.3	— 1.7	2.89	— 0.56666	— 1.2	—1.7×2.47184= —4.202128
0.4	" 1.6	2.56	" 0.53333	" 1.6	" 4.060352
0.5	" 1.5	2.25	" 0.50000	" 2	" 3.89712
0.6	" 1.4	1.96	" 0.46666	" 2.4	" 3.71462
0.7	" 1.3	1.69	" 0.43333	" 2.8	" 3.51481
0.8	" 1.2	1.44	" 0.40000	" 3.2	" 3.29942
0.9	" 1.1	1.21	" 0.36666	" 3.6	" 3.0701
1	" 1	1	" 0.33333	" 4.0	" 2.8284
1.1	" 0.9	0.81	" 0.30000	" 4.4	" 2.57762
1.2	" 0.8	0.64	" 0.26666	" 4.8	" 2.31312
1.3	" 0.7	0.49	" 0.23333	" 5.2	" 2.04204
1.4	" 0.6	0.36	" 0.20000	" 5.6	" 1.76364
1.5	" 0.5	0.25	" 0.16666	" 6.0	" 1.4790
1.6	" 0.4	0.16	" 0.13333	" 6.4	" 1.18928
1.7	" 0.3	0.09	" 0.10000	" 6.8	" 0.89547
1.8	" 0.2	0.04	" 0.06666	" 7.2	" 0.59866
1.9	" 0.1	0.01	" 0.03333	" 7.6	" 0.299838
2	0	0	0	8	0

arc sen $\frac{(a-2)}{3}$	Suma de las tres últimas cantidades	Constante	Superficies
16.274169 π	45.724773	— 45.638203	0.08657 $\frac{r^2}{2}$
16.38837 »	45.825351	45.638203	0.187148 $\frac{r^2}{2}$
16.5 »	45.93928	45.638203	0.301077 $\frac{r^2}{2}$
16.60833 »	46 0621095	45.638203	0.423906 $\frac{r^2}{2}$
16.71583 »	46.199641	45.638203	0.561438 $\frac{r^2}{2}$
16.82083 »	46.3449	45.638203	0.706696 $\frac{r^2}{2}$
16.92500 »	46.50148	45.638203	0.863277 $\frac{r^2}{2}$
17.02667 »	46.662586	45.638203	1.024383 $\frac{r^2}{2}$
17.12750 »	46.830134	45·638203	1.191931 $\frac{r^2}{2}$
17.22667 »	47.006186	45.638203	1.367983 $\frac{r^2}{2}$
17.325 »	47.18618	45.638203	1.547977 $\frac{r^2}{2}$
17.42333 »	48.373493	45.638203	1.735290 $\frac{r^2}{2}$
17.52000 »	47.561832	45.638203	1.923629 $\frac{r^2}{2}$
17.61667 »	47.75525	45.638203	2.117047 $\frac{r^2}{2}$
17.71333 »	47.952727	45.638203	2.314524 $\frac{r^2}{2}$
17.80917 »	48.150628	45.638203	2.512425 $\frac{r^2}{2}$
17.90417 »	48.348248	45.638203	2.710045 $\frac{r^2}{2}$
18 »	48.5488	45.638203	2.910597 $\frac{r^2}{2}$

c) I en el tercer caso

$$S_{2r}^{x} = \frac{x-2r}{2} \sqrt{r^2-(x-2r)^2} + \frac{r^2}{2} \text{arc sen} \frac{x-2r}{2}$$

aplicable desde $x=2r$ hasta $x=3r$. Poniendo $x=ar$

$$S_2^a = \frac{r^2}{2}\left\{(a-2)\sqrt{1-(a-2)^2}+\text{arc sen }(a-2)\right\}$$

Esta fórmula es la misma que la que corresponde al tercer caso del primer perfil i el cuadro que sigue dando las superficies desde 2r hasta 3r es tambien el mismo.

a	$a-2$	$(a-2)^2$	$(a-2)\sqrt{1-(a-2)^2}$	arc sen $(a-2)$	Superficies
2.1	0.1	0.01	0.099499	0.1001542	$0.0998266r^2$
2.2	0.2	0.04	0.19596	0.2013577	$0.1986589r^2$
2.3	0.3	0.09	0.285182	0.3046944	$0.2954382r^2$
2.4	0.4	0.16	0.366608	0.4115182	$0.3890631r^2$
2.5	0.5	0.25	0.433015	0.5235979	$0.4783065r^2$
2.6	0.6	0.36	0.48000	0.6435034	$0.5617517r^2$
2.7	0.7	0.49	0.499898	0.7753971	$0.6376476r^2$
2.8	0.8	0.64	0.48000	0.9273867	$0.7036933r^2$
2.9	0.9	0.81	0.392301	1.1197715	$0.7560362r^2$
3	1	1	0	1.5708	$0.7854000r^2$

II. PERÍMETROS

a) 1er *caso*

$$P_0^x = \frac{r}{2}\text{arc sen}\frac{x-\frac{r}{2}}{\frac{r}{2}} - \frac{r}{2}\,\frac{3\pi}{2}$$

aplicable desde $x=o$ hasta $x=\frac{1}{5}r$

Poniendo $x=a\,r$

$$P_0^a = 2r\left\{\frac{1}{4}\text{ arc sen }2\left(a-\frac{1}{2}\right)-\frac{3\pi}{8}\right\}$$

al 1.er caso corresponde al cuadro siguiente.

a	$\frac{1}{4}$ arc sen $2\left(a-\frac{1}{2}\right)$	Constante $=-\frac{3}{8}\pi$	Perímetros
0.1	$0{,}4264552\pi$	$-1{,}1781$	$2r\times0{,}1615888$
0,2	$0{,}448795\pi$	$-1{,}1781$	$2r\times0{,}231825$

b) 2.º caso

$$P_{\frac{1}{5}r}^{x}=3r\left(\text{arc sen}\ \frac{x-2r}{3}-1.7952\pi\right)$$

aplicable desde $x=\frac{1}{5}r$ hasta $x=2r$

Poniendo $x=a\,r$

$$P_{\frac{1}{5}}^{a}=3r\left(\text{arc sen}\ \frac{a-2}{3}-1.7952\pi\right)=\frac{r}{3}\left(9\ \text{arc sen}\ \frac{a-2}{3}-16.1568\pi\right)$$

Para el 2.° caso tenemos el cuadro siguiente:

a	9 arc sen $\frac{a-2}{3}$	Constante $= -16.1568\pi$	Perímetros
0.3	51.126929	— 50.758203	0.368726 $\frac{r}{3}$
0.4	51.485703	— 50.758203	0.727500 $\frac{r}{3}$
0.5	51.836400	— 50.758203	1.078197 $\frac{r}{3}$
0.6	52.176749	— 50 758203	1.418546 $\frac{r}{3}$
0.7	52.514451	— 50.758203	1.756248 $\frac{r}{3}$
0.8	52.844329	— 50.758203	2.086126 $\frac{r}{3}$
0.9	53.171580	— 50.758203	2.413377 $\frac{r}{3}$
1	53.490986	— 50.758203	2.732783 $\frac{r}{3}$
1.1	53.807754	— 50.758203	3.049551 $\frac{r}{3}$
1.2	54.119306	— 50.758203	3.361103 $\frac{r}{3}$
1.3	54.428220	— 50.758203	3.670017 $\frac{r}{3}$
1.4	54.737134	— 50.758203	3.978931 $\frac{r}{3}$
1.5	55.040832	— 50.758203	4.281629 $\frac{r}{3}$
1.6	55.344530	— 50.758203	4.586327 $\frac{r}{3}$
1.7	55.648197	— 50.758203	4.889994 $\frac{r}{3}$
1.8	55 949298	— 50.758203	5.191095 $\frac{r}{3}$
1.9	55.248086	— 50.758203	5.489883 $\frac{r}{3}$
2.0	56.548800	— 50.758203	5.790597 $\frac{r}{3}$

3er caso

$$P_{2r}^{x} = r \text{ arc sen } \frac{x-2r}{r}$$

aplicable desde $x = 2r$ hasta $x = 3r$

Poniendo $x = a$ r Tenemos

$$\mathfrak{P}_2^a = r \text{ arc sen } (a - 2)$$

El calculo de los perímetros en este caso dá lugar al cuadro siguiente que es el mismo que el del 1.er perfil en el 3.er caso correspondiente.

a	arc sen $(a-2)$	Perímetros
2.1	0,03188 π	0,1001542 r
2.2	0,064094 »	0.2013577 r
2.3	0.096987 »	0,3046944 r
2.4	0,13099 »	0.4115182 r
2.5	0,166666 »	0,5235979 r
2.6	0.204833 »	0.6435034 r
2.7	0.246816 »	0.7753971 r
2.8	0.295167 »	0,9273867 r
2.9	0.3564335 »	1,1197715 r
3	0,5 »	1,5708 r

Superficies, radios medios i Perímetros

Las tablas que siguen, dan, en el caso del colector tipo de «Berlin» las superficies, radios medios i perímetros, i perimetros por las distintas alturas variando de 0,1r, cuyo radio varia entre
m m
1 i 0.25.

Estas tablas han sido establecidas de un modo idéntico a las del 1.er perfil (tipo de Francfort) i tomando en cuenta todas las consideraciones apuntadas allí.

a	Superficies	Perimetros	Radio medio	Alturas	Radio medio
	S	P	$\frac{S}{P}$	m Radio r=1,00	
	Factores de r²	Factores de r	Factores de r	m	m
0.1	0.04157	0.646355	0.0643145	0.10	0.0643145
0.2	0.111834	0,9273	0.1206018	0.20	0.1206018
0.3	0.198404	1.173118	0.1691254	0.30	0.1691254
0.4	0.298982	1.4123	0.2116987	0.40	0.2116987
0.5	0.412911	1.646098	0.2508423	0.50	0.2508423
0.6	0.535740	1.872998	0.2860334	0.60	0.2860334
0.7	0.673272	2.098132	0.3208912	0.70	0.3208912
0.8	0.818530	2.31805	0.3531115	0.80	0.3531115
0.9	0.975111	2.536218	0.3844744	0.90	0.3844744
1	1.136217	2.749156	0.4132967	1.00	0.4132967
1.1	1.303765	2.960334	0.4404097	1.10	0.4404097
1.2	1.479817	3.168036	0.4671086	1.20	0.4671086
1.3	1.659811	3.373978	0.4919448	1.30	0.4919448
1.4	1.847124	3.57992	0.5159679	1.40	0,5159679
1.5	2.035463	3.78172	0,5382374	1.50	0.5382374
1.6	2.228881	3.984852	0.5593385	1.60	0.5593385
1.7	2.426358	4.187296	0.5794570	1.70	0.5794570
1.8	2.624259	4.38803	0.5980495	1.80	0.5980495
1.9	2.821879	4.587222	0.6151608	1.90	0.6151608
2	3.022431	4.787698	0.6312911	2.00	0 6812911
2.1	3.222084	4.988006	0.6459664	2.10	0.6459664
2.2	3.419749	5.190413	0.6588588	2.20	0.6588588
2.3	3.613307	5.397087	0.6694921	2.30	0.6694921
2.4	3.800557	5.610734	0.6773725	2.40	0.6773725
2.5	3.979044	5.834893	0.6819395	2.50	0.6819395
2.6	4.145934	6.074705	0.6825243	2.60	0.6825243
2.7	4.297726	6.338492	0.6780360	2.70	0.6780360
2.8	4.429818	6.642472	0.6668929	2.80	0.6668929
2.9	4.534503	7.027241	0.645275	2.90	0.6452750
3	4.593231	7.920208	0.5792734	3.00	0.5792734

Alturas	Radio medio	Alturas	Radio medio	Alturas	Radio medio
Radio r=0.90 m		Radio r=0.85 m		Radio r=0.80 m	
m	m	m	m	m	m
0.09	0.05788305	0.085	0.05466733	0.08	0.05145160
0.18	0.1085416	0.17	0.1025115	0.16	0.09648144
0.27	0.1522129	0.255	0.1457566	0.24	0.13550032
0.36	0.1905288	0.340	0.1799439	0.32	0.16955896
0.45	0.2257581	0.425	0.2132159	0.40	0.20067384
0.54	0.2574301	0.510	0.2431284	0.48	0.22882672
0.63	0.2888021	0.595	0.2727575	0.56	0.25671296
0.72	0.3178064	0.680	0.3001448	0.64	0.28248920
0·81	0.3460269	0.765	0.3268032	0.72	0.30757952
0.90	0.3719669	0.850	0 3513021	0.80	0.33063728
0.99	0.3963687	0.935	0.3743482	0,88	0.35232776
1.08	0.4203977	1.020	0.3970423	0,96	0,37368688
1.17	0.4427503	1.105	0.4181531	1.04	0.39355584
1.26	0.4643711	1.190	0,4385727	1.12	0 41277432
1.35	0.4844137	1.275	0.4575018	1.20	0 43058992
1.44	0.5034047	1.260	0.4754373	1,28	0.4474708
1.53	0.5215113	1.445	0,4925385	1.36	0.4635656
1.62	0.5382446	1.530	0.5083421	1.44	0.4784396
1.71	0.5536447	1.615	0.5228867	1.52	0.49212864
1.80	0.568162	1.700	0.5365974	1.60	0.50503288
1.89	0.5813698	1.785	0.5490714	1.68	0.51677312
1.98	0.5920729	1.870	0.5600299	1.76	0 52708704
2.07	0.6025429	1,955	0.5690683	1.84	0.53559368
2.16	0.6096353	2.040	0.5757666	1·92	0.54189800
2.25	0.6137456	2,125	0.5796486	2.00	0.54555160
2.34	0.6142719	2,210	0.5801457	2.08	0.54601944
2.43	0.6102324	2,295	0.5763306	2.16	0.5424288
2.52	0.6002036	2,380	0.5668589	2.24	0.53351432
2.61	0.5807475	2,465	0.5484838	2,32	0.5162200
2,70	0,5213461	2,550	0.4923824	2,40	0.46341872

h	Radio medio	h	Radio medio
r=0.70 m		r=0.60 m	
m	m	m	m
0.07	0.04502015	0.06	0.03858870
0.14	0.08442126	0.12	0.07236108
0.21	0.11838778	0.18	0.10147524
0.28	0.14818909	0.24	0.12701922
0.35	0.17558961	0.30	0.15034538
0.42	0.20022338	0.36	0.17162004
0.49	0.22462384	0.42	0.19253472
0.56	0.24717805	0.48	0.21186690
0.63	0.26913208	0.54	0.23068464
0.70	0.28930769	0.60	0.24797802
0.77	0.30828379	0.66	0.26424582
0.84	0.32597602	0.72	0.28026516
0.91	0.34436136	0.78	0.29516688
0.98	0.36117753	0.84	0.30958074
1.05	0.37676618	0.90	0.32294244
1.12	0.39153695	0.96	0.33530310
1.19	0.40561990	1.02	0.34767420
1.26	0.41833275	1.08	0.35882970
1.33	0.43031256	1.14	0.36910348
1.40	0.44190877	1.20	0.37877466
1.47	0.45217648	1.26	0.38753364
1.54	0.46120116	1.32	0.39531528
1.61	0.46864447	1.38	0.40169526
1.68	0.47416075	1.44	0.40542350
1.75	0.47753765	1.50	0.40916370
1.82	0.47776701	1.56	0.40951458
1.89	0.47462520	1.62	0.40582160
1.96	0.46682503	1.68	0.40013574
2.03	0.45169250	1.74	0.3871650
2.10	0.40549138	1.80	0.34756404

h	Radio medio	h	Radio medio	h	Radio medio
m r=0.50		m r=0.33		m r=0.25	
m	m	m	m	m	m
0.05	0.03215725	0.033	0.02922379	0.025	0.016078625
0.10	0.06030090	0.066	0.03979859	0.050	0.03015045
0.15	0.08456270	0.099	0.05581138	0.075	0.04228135
0.20	0 16584935	0.132	0.0698606	0.100	0.052924675
0.25	0.12542115	0.165	0.08277796	0.125	0.062710575
0.30	0.14301670	0.198	0.09439102	0.150	0.071508350
0.35	0.16044560	0.231	0.1058941	0.175	0.080222800
0.40	0.17555575	0.264	0.11652679	0.200	0.088277875
0.45	0.19223720	0.297	0.12687655	0.225	0.096118600
0.50	0.20664830	0.330	0.136387911	0.250	0.103324175
0.55	0.22020485	0.363	0.1453352	0.275	0.110102425
0.60	0.23355430	0.396	0.15444584	0.300	0.11677715
0.65	0.24597240	0.429	0.16234178	0.325	0.12299752
0.70	0.25798395	0.462	0.17026944	0.350	0.129091975
0.75	0.26911870	0.495	0.17761834	0.375	0.13445935
0 80	0.27966925	0.528	0.18458171	0.400	0.139834625
0.85	0.2897285	9.561	0.19122081	0.425	0.14486425
0.90	0.29902475	0.594	0.197356335	0.450	0.14951238
0.95	0.3075804	0.627	0.20300306	0.475	0.15379020
1.00	0.31564555	0.660	0.20832606	0.500	0.157822775
1.05	0.32298320	0.693	0.213168912	0.525	0.161491600
1.10	0.32942940	0.726	0.2174234	0.550	0.16471470
1.15	0.33471605	0.759	0.22093239	0.575	0.167373025
1.20	0.33868625	0.792	0.22353293	0.600	0.169343125
1.25	0.34096975	0.825	0.22504004	0.625	0.170484875
1.30	0.34126215	0.858	0.22523302	0.650	0.170631075
1.35	0.3390180	0.891	0 22375188	0.675	0.16950900
1.40	0.33344645	0.924	0.22007466	0.700	0.16672323
1.45	0.3226375	0.957	0.21294075	0.725	0.16131875
1.50	0.28963670	0.990	0.19116022	0.750	0.14481835

a	h	Superficies	h	Superficies	h	Superficies
	m radio=1.00		m radio=0.90		m radio=0.85	
	m	m	m	m	m	m
0.1	0.10	0.04157	0.09	0,0336717	0,085	0,0300343
0.2	0.20	0 111834	0.18	0.0905855	0,17	0.08080007
0.3	0.30	0.198404	0.27	0.1607072	0,255	0,1433469
0.4	0.40	0.298982	0.36	0.2421754	0,340	0,2160145
0.5	0.50	0.412911	0.45	0.3344579	0,425	0,2983282
0.6	0.60	0.535740	0.54	0.4339494	0.510	0.3870722
0,7	0.70	0.673272	0.63	0.5453503	0,595	0,48643902
0.8	0.80	0.818530	0.72	0.6630003	0,680	0,5913879
0.9	0.90	0.975111	0.81	0.7898399	0,765	0,7045177
1.0	1.00	1.136217	0.90	0.9203358	0,850	0,82091678
1.1	1.10	1.303765	0.99	1.0560497	0,935	0,9419702
1.2	1.20	1.479817	1.08	1.1986518	1,020	1,06916778
1.3	1,30	1.659811	1.17	1.344447	1,105	1,19921345
1.4	1.40	1.847124	1.26	1.4961704	1.190	1,3345471
1.5	1.50	2.035463	1.35	1.6487250	1,275	1,4706220
1.6	1.60	2.228881	1.44	1.8052936	1,360	1,6103665
1.7	1.70	2.426358	1.53	1.9653499	1,445	1,75304366
1.8	1.80	2.624259	1.62	2.1256498	1.530	1,89602713
1.9	1 90	2.821879	1.71	2.285722	1.615	2,0388076
2.	2.00	3.022431	1.80	2.4481691	1,700	2.1837064
2.1	2·10	3.222084	1.89	2.609888	1,785	2,3279557
2.2	2.20	3.419749	1.98	2.7699967	1,870	2.4707687
2.3	2.30	3.613307	2.07	2.9167787	1,955	2.6106143
2.4	2.40	3.800557	2.16	3.0784612	2.040	2,7459024
2.5	2.50	3.979044	2.25	3.2230256	2,125	2,874859
2.6	2.60	4.145934	2.34	3.3582065	2,210	2,995437
2.7	2.70	4.297726	2.43	3.4811481	2,295	3,10510700
2.8	2.80	4.429818	2.52	3.5881526	2,380	3,2005435
2.9	2.90	4.534503	2.61	3.9729474	2,465	3,2761784
3	3.00	4.593231	2.70	3.7205171	2.550	3,3186094

h	superficies	h	superficies	h	superficies
radio=0.80 m		radio=0.70 m		radio=0.60 m	
m	m	m	m	m	m
0.08	0.0266048	0.07	0.0203693	0.06	0.0149652
0.16	0.0715738	0.14	0.0547987	0.12	0.0402602
0.24	0.1269786	0.21	0.0972179	0.18	0.0714254
0.32	0.1913485	0.28	0.1465012	0.24	0.1076335
0.40	0.264233	0.35	0.2023264	0.30	0.1486479
0.48	0.3428736	0.42	0.2625126	0.36	0.1928664
0.56	0.4308941	0.49	0.3299033	0.42	0.2423779
0.64	0.523859	0.56	0.4010797	0.48	0.2946708
0.72	0.624071	0.63	0.4778044	0.54	0.35103996
0.80	0.7271780	0.70	0.5567463	0.60	0.4090381
0.88	0.8344096	0.77	0.63884485	0.66	0.4704354
0.96	0.9470829	0.84	0.7251103	0.72	0.53273412
1.04	1.062279	0.91	0.8133074	0.78	0.5975319
1.12	1.1821594	0.98	0.9050908	0.84	0.6649646
1.20	1.3026963	1.05	0.9973769	0.90	0.7327667
1.28	1.4264838	1.12	1.0921517	0.96	0.80239716
1.36	1.5528691	1.19	1.1889154	1.02	0.8734889
1.44	1.6795258	1.26	1.2858859	1.08	0.94473324
1.52	1.8060026	1.33	1.382721	1.14	1.0458764
1.60	1.9343558	1.40	1.4809912	1.20	1.0880752
1.68	2.0321338	1.47	1.5788212	1.26	1.1599502
1.76	2.1886394	1.54	1.6756770	1.32	1.23110964
1.84	2.3125165	1.61	1.7705204	1.38	1.3007905
1.92	2.4323565	1.68	1.8622729	1.44	1.3682005
2.00	2.5465882	1.75	1.949732	1.50	1.4324558
2.08	2.6533978	1.82	2.0315077	1.56	1.4925362
2.16	2.7505446	1.89	2.1058857	1.62	1.5471814
2.24	2.8350835	1.96	2.1706108	1.68	1.5947345
2.32	2.9020819	2.03	2.2219065	1.74	1.6324211
2.40	2.9396678	2.10	2.2506832	1.80	1.6535632

h	Superficies	h	Superficies	h	Superficies
r=0.50 m		r=0.33 m		r=0.25 m	
m	m	m	m	m	m
0.05	0.0103925	0.033	0.00452697	0.025	0.00259813
0.10	0.0279585	0.066	0.01217872	0.050	0.00698963
0,15	0.0496010	0.099	0.02160619	0.075	0.01240025
0.20	0.0747455	0.132	0.0325591	0.100	0.0186864
0.25	0.1032277	0.165	0.0449660	0.125	0,02580694
0.30	0.133935	0.198	0.0583421	0.150	0.0334838
0.35	0.168318	0.231	0.07331932	0.175	0.0420795
0.40	0.2046325	0.264	0.0891379	0.200	0.05115813
0.45	0.2437778	0.297	0.1061896	0.225	0.0609444
0.50	0.284054	0.330	0.12373403	0.250	0.07101356
0.55	0.325941	0.363	0.1419800	0.275	0.0814853
0.60	0.369954	0.396	0.1611521	0.300	0.0924886
0.65	0.414953	0.429	0.1807534	0.325	0.1037382
0.70	0.461781	0.462	0.2011518	0.350	0.1154453
0.75	0.5088658	0.495	0.2216619	0.375	0.1272164
0.80	0.557220	0.528	0.2427251	0,400	0.1393051
0.85	0.606589	0.561	0.2642304	0.425	0.1516474
0.90	0.6560648	0.594	0.285782	0.450	0.1601619
0.95	0.7054697	0.627	0.3073026	0.475	0.1763674
1.00	0.7555078	0.660	0.3294427	0.500	0.1889019
1.05	0.8055210	0.693	0.3508849	0.525	0.2013803
1.10	0.854937	0.726	0.3724107	0.550	0.2137343
1.15	0.9033258	0.759	0.3934891	0.575	0.22583168
1.20	0.9501393	0.792	0.4138807	0.600	0.23753481
1.25	0.994761	0.825	0.4333179	0.625	0.2486903
1.30	1.0364835	0.858	0.4514922	0.650	0.2591209
1.35	1.0744315	0.891	0.4680224	0.675	0.2686079
1.40	1.107455	0.924	0.4824072	0.700	0.2768636
1.45	1.1336258	0.957	0,4938074	0.725	0.2834064
1.50	1.148308	0.990	0.5002029	0.750	0.2870769

h	Perimetros	h	Perímetros	h	Perímetros
r=1.00 m		r=0.90 m		r=0.85 m	
m	m	m	m	m	m
0.10	0.646355	0.09	0.5817195	0.085	0.5494018
0.20	0,9273	0.18	0.8345700	0.17	0.788205
0;30	1.173118	0.27	1.0558062	0.255	0.9971503
0.40	1.41230	0.36	1.271070	0.340	1.200455
0.50	1.646098	0.45	1.4814882	0.425	1.3991833
0.60	1.872998	0.54	1.6856982	0.510	1.5910483
0.70	2.098132	0.63	1.8883188	0.595	1.7834122
0.80	2.31805	0.72	2.086245	0.680	1.9703425
0,90	2.536218	0.81	2.2825962	0.765	2.1557853
1.00	2.749156	0.90	2.4742404	0.853	2.3367826
1.10	2.960334	0.98	2.6643006	0.935	2.5162839
1.20	3.168036	1.08	2.8512324	1.020	2.6928306
1.30	3.373978	1.17	3.0365802	1.105	2.8678793
1.40	3,57992	1.26	3,221928	1.190	3.0429320
1.50	3.78172	1.35	3.403548	1.275	3.214462
1.60	3.984852	1:44	3.5863668	1.360	3.3871242
1.70	4.187296	1.53	3.7685664	1.445	3.5578016
1.80	4.38803	1.62	3.949227	1.530	3.7298255
1.90	4.587224	1.71	4.128498	1.615	3.8991387
2.00	4.787698	1.80	4.3089282	1.700	4.0695433
2.10	4.988003	1.89	4.4892054	1.785	4.2398051
2.20	5.190413	1.98	4.6713717	1.870	4.4118511
2.30	5.397087	2.07	4.8573783	1.955	4.5875239
2.40	5.610734	2.16	5.0498606	2.040	4.7691239
2.50	5.834893	2.25	5.2514037	2.125	4.9596591
2.60	6.074705	2.34	5.4672345	2.210	5.1634993
2.70	6.338492	2.43	5.7046428	2.295	5.3876882
2.80	6.642472	2.52	5.9782248	2.380	5.6461012
2,90	7.027241	2.61	6.3244169	2.465	5.9731549
3.00	7.929298	2.70	7.1363682	2.550	6.7419033

h	Perímetros	h	Perímetros	h	Perímetros
	m r=0.80		m r=0.70		m r=0,60
m	m	m	m	m	m
0.08	0.5170840	0.07	0.4524485	0.06	0.3878130
0.16	0.74184	0.14	0.649110	0.12	0.556380
0.24	0,9384944	0.21	0.8211826	0.18	0.7038708
0.32	1.12984	0.28	0.988610	0.24	0.847380
0.40	1.3168784	0.35	1.1522686	0.30	0.9888588
0.48	1.4983984	0.42	1.3110986	0.36	1.1237988
0.56	1.6785056	0.49	1.4686924	0.42	1.2588792
0.64	1.854440	0.56	1.622635	0.48	1.390830
0.72	2.0289744	0.63	1.7753526	0.54	1.5217308
0.80	2.1993248	0.70	1.9244092	0.60	1.6494936
0.88	2.3682672	0.77	2.0722338	0.66	1.7762004
0.96	2.5344288	0 84	2.2176252	0.72	1.9008216
1.04	2.6991824	0,91	2.3617846	0.78	2.0243868
1·12	2.863936	0.98	2.505944	0.84	2.147952
1.20	3.025376	1.05	2.647204	0.90	2.269032
1.28	3.1878816	1.12	2.7893964	0.96	2.3909112
1.36	3.3498368	1.19	2.9311072	1.02	2.5123776
1.44	3·510424	1.26	3.071621	1.08	2.632818
1.52	3.669776	1.33	3.211054	1.14	2.752332
1.60	3.8301584	1.40	3,3513886	1.20	2.8726188
1.68	3.9904048	1.47	3.4916042	1.26	2.9928036
1.76	4.1523304	1.54	3.6332891	1.32	3.1142478
1.84	4.3176696	1.61	3.7779609	1.38	3.2382522
1,92	4.4885872	1.68	3.9275138	1.44	3.3664404
2.00	4.6679144	1.75	4.0844251	1 50	3.5009358
2.08	4.8597640	1.82	4.2522935	1.56	3·6448230
2.16	5.0707936	1.89	4.4369444	1.62	3.8030952
2.24	5 3139776	1.96	4.6497304	1.68	3.9854832
2.32	5.6217928	2.03	4.9190687	1.74	4.2163446
2.40	6.3434384	2,10	5.5505086	1.80	4.7575788

h	Perímetros	h	Perímetros	h	Perímetros
m r=0.50		m r=0.33		m r=0,25	
m	m	m	m	m	m
0.05	0.3231775	0.033	0.2132972	0.025	0.16158875
0.10	0.46365	0.066	0,306009	0.050	0.231825
0.15	0.585559	0.099	0.3871289	0.075	0.2932795
0.20	0.70615	0.132	0.466059	0.100	0.353075
0.25	0.823049	0.165	0.5432123	0.125	0.4115245
0.30	0.936499	0.198	0.6180893	0.150	0.4682495
0,35	1.049066	0.231	0.6923836	0.175	0.524533
0.40	1.159025	0.264	0.7649565	0.200	0.5795125
0.45	1.268109	0.297	0.8369519	0.225	0.6340545
0.50	1.374578	0.330	0.9072215	0.250	0.687289
0.55	1.480167	0.363	0.97694022	0.275	0.7400835
0.60	1.584018	0.396	1.0454519	0,300	0.792009
0.65	1.585989	0.429	1.1134127	0.325	0.8434945
0,70	1.78993	0.462	1.1813736	0.350	0.89498
0.75	1.89086	0.495	1.247968	0.375	0.94543
0.80	1.992426	0.528	1.3150012	0.400	0,996213
0 85	2.093648	0.561	1.3818077	0 425	1.046824
0.90	2.194015	0.594	1.4480499	0.450	1.0970075
0.95	2.29361	0.627	1.51378326	0.475	1.146805
1.00	2.393849	0.660	1.5799403	0.500	1.1969245
1,05	2.494003	0.693	1.6460419	0.525	1.2470015
1.10	2.5952065	0.726	1.7128363	0.550	1.29760325
1.15	2.6985435	0.759	1.7810387	0.575	1.34927175
1.20	2.805367	0.792	1.8515422	0.600	1.4026835
1.25	2.9174465	0.825	1.9255147	0.625	1.45872325
1.30	3.0373525	0 858	2.0046527	0.650	1.51867625
1.35	3.169246	0.891	2.0917024	0.675	1.584623
1.40	3.321236	0.924	2.1920158	0.700	1.660618
1.45	3.513621	0.957	2:3189895	0.725	1.7568105
1.50	3.964649	0.990	2.6166683	0.750	1.9823245

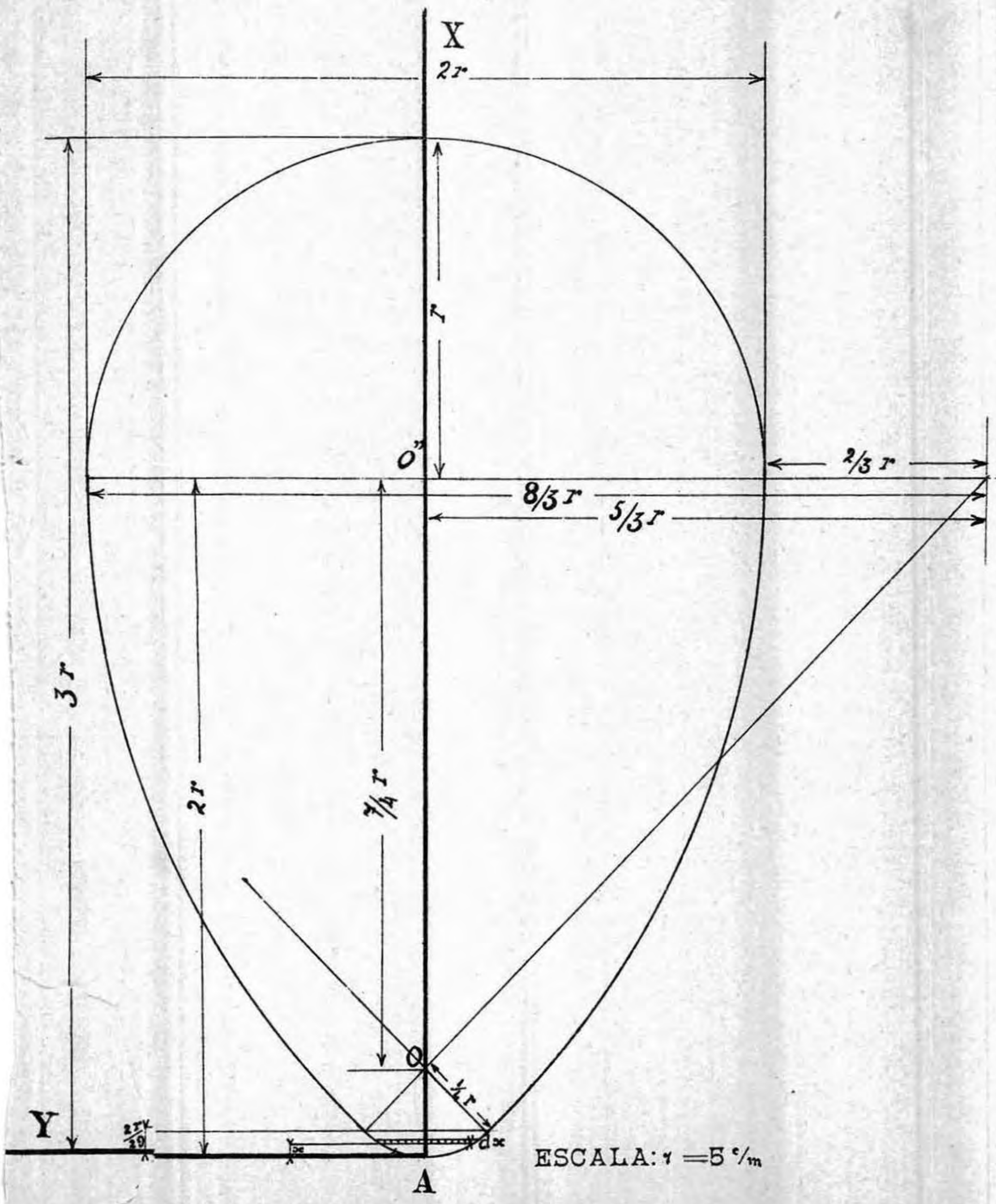
N. B:—Para la aplicacion de estas tablas correspondientes al tipo de colector de Berlin se seguirá un procedimiento idéntico al del primer perfil (Francfort).

Santiago, Mayo de 1897.

FR. DE SUTTER.

TIPO DE FRANCFORT

Fig. 1



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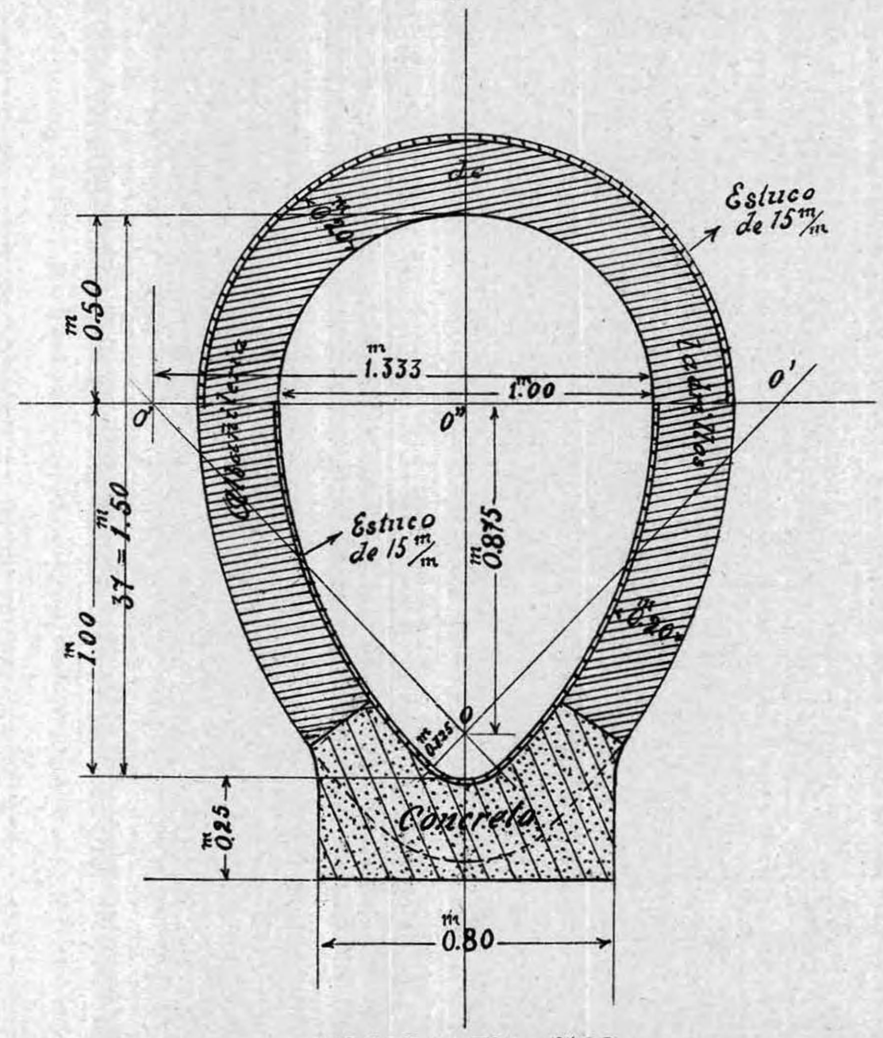
COLECTOR OVOIDE DE FRANCFORT

DE 1.m50 × 1.m00 DE SECCION

Fig. 2



ESCALA: 5 c/m = 1.m00

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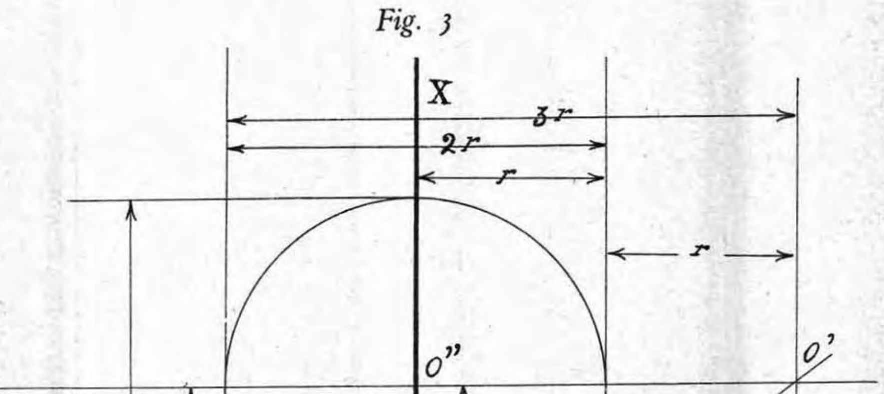
TIPO DE COLECTOR DE BERLIN

Fig. 3



ESCALA: 5 c/m = 1.m 00

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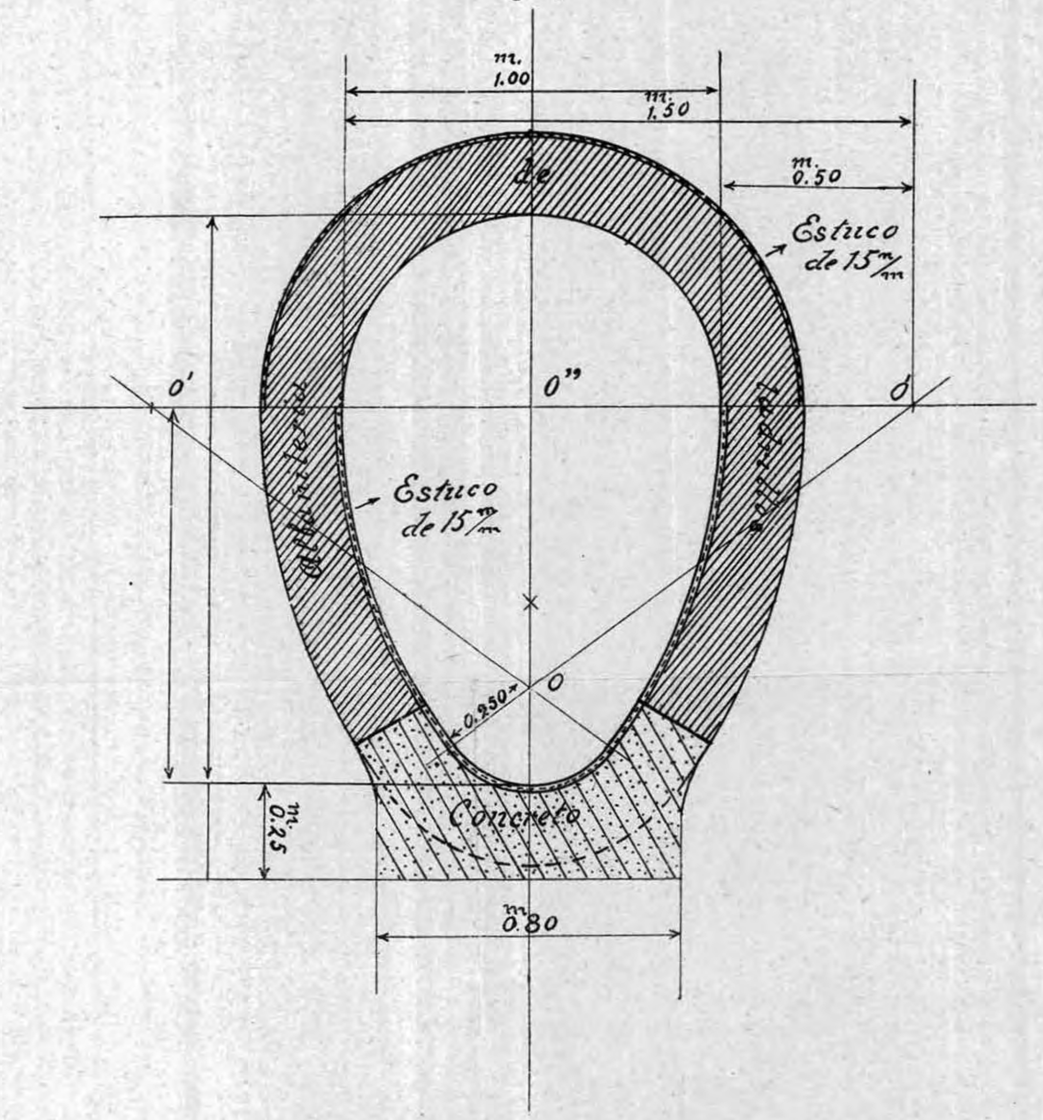
COLECTOR OVOIDE TIPO DE BERLIN

DE 1.m50 × 1.m00 DE SECCION

Fig. 4



ESCALA: 5 c/m = 1.m00

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